Title: **PHOENICS Predictions of Large Amplitude Internal Waves in the Ocean**

Authors: Dr R P Hornby and Mr R J Small, Underwater Sensors and Oceanography Department, Building B21, Winfrith Technology Centre, Winfrith Newburgh, Dorset, DT2 8XJ.

Tel: +44 (0) 1305 212325; Fax: +44 (0) 01305 212950

Email: RPHORNBY@mail.dera.gov.uk.

Date: January 2000

Computer: Silicon Graphics Power Challenge running under IRIX v 6.5.4

PHOENICS: Version 3.2

ABSTRACT

Internal waves are ubiquitous in the ocean and result from disturbances to the stable ocean stratification. Such disturbances may arise from a variety of sources including wind stress fluctuations, variation in atmospheric pressure fields, heavy rain, turbulence, wave action or flow over topography (for example tidal flows over the shelf edge in stratified waters may generate internal waves - the so called internal tides). In particular, large amplitude internal waves (which may evolve into isolated waves of fixed form called solitons) are commonly produced at continental shelf breaks (and other local topographic features) by tidal forcing. These waves have amplitudes of order tens of metres and orbital velocities of order tens of cm/s and therefore significantly affect the local stratification and current shear.

These waves are important from the military point of view because they affect the propagation of sound in the surface layers of the ocean. Hence the propagation of these waves and the way that they interact with each other need to be predicted in order to make an assessment of their effect on acoustic sensors. The currents associated with large amplitude internal waves may have a dramatic effect on moored offshore platforms if their presence is not anticipated and the enhancement of local mixing due to the waves is also important in understanding the distribution of nutrients and pollutants in shelf-edge seas. Because of the inherent non-linearity, it is not possible to derive analytical solutions of general applicability and so numerical methods need to be applied.

This paper describes some of the ongoing work conducted at the Defence Evaluation and Research Agency (DERA) in assessing the potential of the Computational Fluid Dynamics code PHOENICS to predict the propagation and interaction of large amplitude internal waves. This includes use of sea trial thermistor chain data and Synthetic Aperture Radar imagery of the ocean surface.

CONTENTS LIST

OBJECTIVES OF WORK

Internal waves are found everywhere in the stratified oceans of the world [1]. They arise due to disturbances induced in the essentially stable ocean stratification by wind fluctuations, variation in atmospheric pressure fields, heavy rain, turbulence, wave action or flow over topography. In particular, large amplitude internal waves (which may evolve into isolated waves of fixed form called solitons) are commonly produced at continental shelf breaks (and other local topographic features) by tidal forcing. These waves have amplitudes of order tens of metres and orbital velocities of order tens of cm/s and therefore significantly affect the local stratification and current shear.

Fig 1a shows a Synthetic Aperture Radar (SAR) image of the sea surface at the Malin Shelf (north west of Ireland) taken from the European (Space Agency) Remote Sensing satellite (ERS-1) during the summer of 1995. The dotted lines are lines of constant depth superimposed on the SAR image. The 'ripples' appearing in the image are surface manifestations of large amplitude internal waves moving from deep to shallow water. Fig 1b shows a contour plot of temperature measurements from a high resolution thermistor chain deployed by the Defence Evaluation and Research Agency (DERA) in the same area. The vertical axis is depth in metres and the horizontal axis time in units of hours. A large amplitude internal wave followed by three smaller waves can be seen from this record. Fig 1c shows the SAR surface image of a similar wave train. These waves cause a significant disturbance to the local temperature structure (the leading wave, for example, causes a 40m to 50m downward penetration of warmer water). The example here shows internal waves of depression; internal waves of elevation are also possible.

Since the speed of propagation of sound in the ocean is a function of temperature, salinity and pressure, such large disturbances to the ocean stratification will affect the way sound propagates through the ocean and hence the performance of acoustic sensors. In addition the waves induce pulses of current which can affect submerged vehicles or diving operations..

The objective of the current work is to assess the capability of the Computational Fluid Dynamics (CFD) code PHOENICS in predicting the evolution and interaction of large amplitude internal waves with other large amplitude internal waves and with bathymetry. The results will allow operational knowledge to be gained on the associated variability of density stratification and current shear in the ocean.

DESCRIPTION OF WORK

Just as a disturbance on a water/air surface causes a surface wave which propagates away from the disturbance at a finite speed so a disturbance in the interior density stratification of the ocean causes a propagating internal wave. These waves propagate at much smaller speeds (~1-2m/s maximum) than their surface counterparts (because of the smaller, driving density differential) and have periods typically ranging from minutes to many hours. Internal waves with small amplitudes can be treated with linear theory, but for the larger amplitude waves non-linear effects are important. Non-linear effects tend to cause a steepening of the wave whereas dispersive and (the smaller) dissipative effects cause the wave to broaden. If a balance is achieved between these two competing effects then the wave propagates unchanged (the solitary wave).

In general internal waves may propagate through the ocean depths, and are not confined only to the ocean pycnocline (the region of largest density variation). However, the large amplitude internal waves discussed here do propagate along the pycnocline. These waves may have

amplitudes upto 100m, with horizontal scales of many hundreds of metres and phase speeds upto 2m/s. The orbital velocities associated with the larger amplitude of these waves may be as large as 1m/s.

Weak non-linear effects may be described mathematically ([2], [3] and [4]) using perturbation series about the linear state. If the perturbation parameter is chosen equal to the ratio of wave amplitude to water depth then analysis results in the familiar Korteweg de Vries (KdV) equation which includes first order non-linear corrections and the Fully Extended Korteweg de Vries (FEKdV) equation which includes first and second order corrections. The choice of a different perturbation parameter (see [2]) results in the Extended Korteweg de Vries (EKdV) equation which also includes first and second order non-linear corrections but which has more general applicability to large amplitude waves (and which may be applicable for a range of wave amplitudes from small to large under certain circumstances [2]). These equations can be solved in closed form for special cases and can be integrated very quickly with high accuracy for more general cases, but require increasing sophistication and modification to deal with interacting waves in three dimensions, generalised stratification and variable bathymetry.

The EKdV equation for the displacement, η, of the interface at range position x and time t in a two layer system is [2],

$$\frac{\partial \eta}{\partial t} + (c + \alpha_0 \eta - \alpha_1 \eta^2)\frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0$$

Where c, α_0, α_1 and β are constants (see [2] for details). With $\alpha_1 = 0$ this reduces to the KdV equation.

This equation has a solitary wave solution of amplitude a of the form

$$\eta(x,t) = a\frac{\sec h^2[\kappa(x - C_m t)]}{1 - \mu \tanh^2[\kappa(x - C_m t)]}$$

Where κ, μ and C_m are constants (see [2] for details).

The present analysis utilises PHOENICS to solve the full non-linear equations of motion which (using the standard notation) are,

$$\frac{\partial(\rho\phi)}{\partial t} + div(\rho u\phi) = div(\Gamma_\phi \nabla \phi) + S_\phi$$

Where ϕ is 1 for mass conservation, u, v or w for momentum conservation and h for energy conservation. These equations are solved for generalised stratification and arbitrary bathymetry in 2-D or 3-D cartesian coordinates. Initial waveforms are prescribed in the domain either from solitary wave solutions to the perturbed equations of motion or from measured waveforms in the ocean. The ocean surface is represented as a rigid lid to good approximation since the surface elevations induced by internal waves are small compared to the internal wave amplitude. The lateral boundaries may be either solid, cyclic (*x*-direction), or inflow or outflow (fixed pressure). In particular, the inflow boundary condition is used to render travelling waves stationary or near stationary by applying an inflow velocity equal to or approximating the wave phase speed.

The analysis in this paper treats the ocean surface and ocean bottom as free-slip boundaries although provision has already been included in the Satellite and Ground modules for non-linear surface and bottom stresses. Similarly, although coding has been included for turbulence modelling using the k, ε model with buoyancy correction, results are presented here assuming that the flow is laminar (diffusive effects are generally considered to be smaller than dispersive effects).

PHOENICS SETTINGS

The settings below are the main settings for a 2-D case with the x axis horizontal and the y axis vertical. 3-D cases have similar settings with the z axis in the vertical and the x and y axes running horizontally, though the spatial grids are necessarily much coarser (40m in the horizontal and 5m in the vertical directions).

Satellite

Group 1; set TITLE, various REAL, INTEGER, BOOLEAN constants.

Group 2; set STEADY=F, set time step typically 20s.

Group 3; set CARTES=T, set x lateral grid cell typically 10m.

Group 4; set y vertical grid cell typically 1m.

Group 7; Solve(P1,U1,V1,H1); Store(TMP1,PRPS,DEN1).

If x-direction cyclic boundary required:

Group 8; XCYCLE=T.

Seawater properties:

Group 9; RHO1=GRND1; TMP1=GRND2

Initial wave form read into Ground; define bathymetry (fixed gradient slope); lateral boundaries:

Group 11; INIVAL patch type for U1 and H1 with GRND value;

INIVAL patch type for PRPS with coefficient of 0.0 and value 199;

Patches as required for lateral (x-direction) boundaries, with pressure specified in Ground and (constant) U1 velocity specified explicitly for inflow boundary.

High order spatial discretisation scheme; full density with reduced pressure:

Group 13; Scheme(KOREN,ALL);

BUOYA=0.0; BUOYB=-9.81; BUOYC=0.0; BUOYD=mean seawater density

Patch(BUOY,PHASEM,1,NX,1,NY,1,NZ,1,#NREGT)

Coval(BUOY,V1,FIXFLU,GRND2).

Group 15; LSWEEP=30.

Group 17; LINRLX of 0.7 for P1; FALSDT of 500*XULAST/NX for U1,V1,H1.

Ground

Group 1; read in initial density and U1 velocity fields. Note that the U1 velocity field initiates the direction of propagation of a wave with a symmetric profile. Map density to H1 and calculate vertical pressure profile for fixed pressure lateral boundary.

Group 11; set F array VALUES for U1 and H1 from Group 1 input data.

Group 13 (section 12); set F array VALUES for P1 from Group 1 pressure calculations.

Group 19 (section 8); include coding to pick out mean density contour of wave for output.

PRESENTATION OF RESULTS

In order to assess the capability of PHOENICS a number of cases were considered:

1. Propagation of small and large amplitude 2-D solitary internal waves in a two layer fluid and comparison with analytical solutions.

2. Interaction of colliding 2-D large amplitude internal waves in a two layer fluid.

3. Propagation of 2-D small and large amplitude internal waves up a slope in a two layer fluid.

4. Propagation of 2-D large amplitude internal wave up a slope and impingement on slope in a two layer fluid.

5. Propagation and interaction of 3-D large amplitude internal waves in a continuously stratified fluid.

6. Propagation and interaction of 3-D large amplitude internal waves in a continuously stratified fluid with variable bathymetry.

   Results from these cases are presented in figs 2 to 8. The water depth for all the 2-D cases is 140m with the lower layer occupying a depth of 90m (density 1027.26 kg/m^3) and the upper layer a depth of 50m (density 1026.73 kg/m^3). These parameters are chosen so that the EKdV model gives good results for both the small amplitude and large amplitude internal waves [2]. The cell sizes in the x (horizontal) and y (vertical) directions for the 2-D cases are respectively 10m and 1m and the time step is 20s. The slope gradient used in cases 3 and 4 is 0.05 which is much larger than most continental slopes in the ocean (which generally have slopes <<0.1) and more characteristic of the slopes in lakes but which is used here in assessing extremes in capability.

   For the 3-D cases the fluid is continuously stratified and the water depth is 140m. The density stratification is representative of the Malin Shelf region in summer. The cell sizes in the x and y directions are 40m. The cell size in the z (vertical) direction is 5m. The time step is 10s.

   Initial tests were performed on a large amplitude internal wave propagating in a two layer fluid using the UPWIND, SMART, KOREN and UMIST numerical schemes. The UPWIND scheme was clearly more diffusive than the latter three schemes despite a seemingly adequate resolution of the wave on the grid. However, there was little to choose between the latter three schemes and KOREN was selected because favourable experience had already been gained with the scheme in solving other problems. Satisfactory convergence was then achieved for all the cases considered using the spatial and temporal discretisations and relaxation specified under PHOENICS SETTINGS. Grid independence has, so far, only been assessed for the first case and is discussed below. Computer storage and time for execution is only a serious problem with the current computing arrangements for the 3-D cases and has necessitated the use of a smaller domain and coarser spatial resolution for these cases (see above).

DISCUSSION

The first case considers the propagation of 5m and 18m amplitude internal solitary waves. For the 5m wave, the KdV, FEKdV and EKdV solutions are essentially the same and the wave as it propagates is expected to retain the shape determined by the solution of these equations. Fig 2a shows the initial waveform (the solution of the KdV equation) used in the PHOENICS simulation. Cyclic boundary conditions are applied in the x direction. Fig 2b shows the waveform 6000s later in the simulation. It can be seen that although the wave amplitude has decreased from 5m to 4.3m the shape of the wave is still well represented by the soliton solutions. The amplitude decrease for this case is perhaps not too disappointing bearing in mind the fact that with a 1m vertical cell size there are only 5 cells resolving the wave shape in the vertical.

For the 18m wave, the best solution is anticipated from the EKdV model and the KdV and FEKdV solutions are expected to be inaccurate [2]. To test this out, the PHOENICS simulation is initialised with the EKdV solitary wave solution shown in fig 3a. It can be seen that this solution differs considerably from the KdV and FEKdV solutions. For this case an inflow boundary was used on the east boundary (towards which the wave propagates) and a fixed pressure boundary on the west face boundary downstream of the wave. The inflow velocity was chosen to be the phase speed of the wave (0.4m/s) from the EKdV solution. Fig 3b shows the results of the simulation after 10000s. The PHOENICS results are shown to be in very good agreement with the EKdV solitary wave solution. For this case there is good vertical resolution of the wave and consequently very little reduction in the wave amplitude. Also the simulation shows no movement of the wave, confirming that PHOENICS reproduces the correct phase speed.

Grid sensitivity tests were carried on the above 18m wave propagation case. Results with halving the time step to 10s showed no significant change, but oscillations started to appear when a time step of 40s was used. Increasing the vertical grid cell size from 1m to 2m and then to 5m showed negligible change in results. This was also the case when the horizontal grid cell size was increased from 10m to 40m in combination with the changes in the vertical grid size.

The results from the second case simulation are shown in figs 4a to 4d. Here the 2-D domain (with cyclic east/west boundary conditions) is initialised with two 20m EKdV solitary waves with U1 velocity fields for each wave chosen so that the waves move towards each other. This is the situation shown in fig 4a. Fig 4b shows the waves interacting with an approximate doubling of the wave amplitude. Fig 4c shows the waves emerging from the interaction and fig 4d shows the waves again as separate entities (with approximately the same amplitude as the original waves) travelling away from each other. The broadening of the contours in the vicinity of the interaction region indicates an enhanced mixing effect there.

Figs 5a to 5d show the effect of introducing bathymetry in the form of a slope and constant depth shelf (case 3). Figs 5a and 5b illustrate the effect of a 5m amplitude KdV solitary wave travelling up the slope. The remaining figures show the effect on a much larger amplitude 20m EKdV solitary wave. The steepening effect on the wave due to the reduction in phase speed with depth is clearly shown in each case, though for the larger amplitude wave it is much more severe as the wave interacts more directly with the bathymetry (the wave is about to break).

The direct interaction of a large amplitude internal wave with the bottom slope is shown in more detail in figs 6a to 6d (case 4). Here a 20m amplitude EKdV solitary wave is shown

breaking on the slope (figs 6c). The downwash from the slope and a strong vortex at the base of the wave are clearly indicated in the velocity vector plot of fig 6d (corresponding to the wave position in fig 6b). The waves seen on the outer edge of the downwash layer are probably due to the non-uniform stepped nature of the slope rather than some interfacial instability. The characteristics of this simulation are qualitatively similar to the experiments reported in [5].

Fig 7a shows a SAR sea surface image of the Malin Shelf area. Waves marked as B1 and C1 moving shorewards have interacted to form a third stronger wave A1. A 3-D PHOENICS simulation (case 5) has been set up in order to assess the capability of dealing with interacting waves of the type shown in fig7a. A rectangular domain with solid free slip boundaries has been initialised with cylindrical waves (taken from the solitary wave solution to the KDV equation) with origin along the south east and south west corners of the domain. This initial situation is shown in fig 7b where the sides of the domain are contoured with the density and the surface is contoured with the V1 velocity (the surface velocity magnitude has been used in later simulations). The waves propagate out towards the centre of the domain where a wave-wave interaction takes place (qualitatively similar to the SAR image). Note the rapid decay of the waves due to radial spreading. Also there are some small errors which cause some asymmetry in the simulation (fig 7e surface plot).

Figs 8a and 8b show the same simulation but over an axisymmetric sinusoidal seamount of maximum height h having the form $h*sin(\pi x/XULAST)*sin(\pi y/YVLAST)$ (case 6). In this case the surface pressure has been used to trace the wave propagation. Fig 8a shows the retardation of the waves towards the centre of the domain as they pass over the shallowest depth (in comparison to the waves travelling along the east and west boundaries of the domain where the water is deeper). Fig 8b also shows the wave steepening taking place over the central section of the seamount.

CONCLUSIONS

This work constitutes the first stage of an ongoing assessment of the capability of the PHOENICS code in representing the ocean structure associated with the propagation and interaction of large amplitude internal waves. Six cases have been defined to form an initial basis for testing this capability. Very good comparison with available theory has been obtained for the first case involving propagation of small and large amplitude solitary waves in a two layer fluid. The remaining cases are more complicated and are not as amenable to a simple closed form solution but the simulations appear physically plausible and realistic and encourage further investigation using PHOENICS.

RECOMMENDATIONS

Further work will be carried out using PHOENICS with particular attention being paid to comparison with available theory, experiment and DERA trials for internal wave-wave interaction and internal wave-bathymetry interaction. Attention will be directed towards a smoother representation of bathymetric features using a partial porosity treatment or body-fitted grid. The possible use of a higher order temporal integration scheme will be also be explored.

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