Simulation of Free Surface and Interfacial Flows Using a Level Set Method

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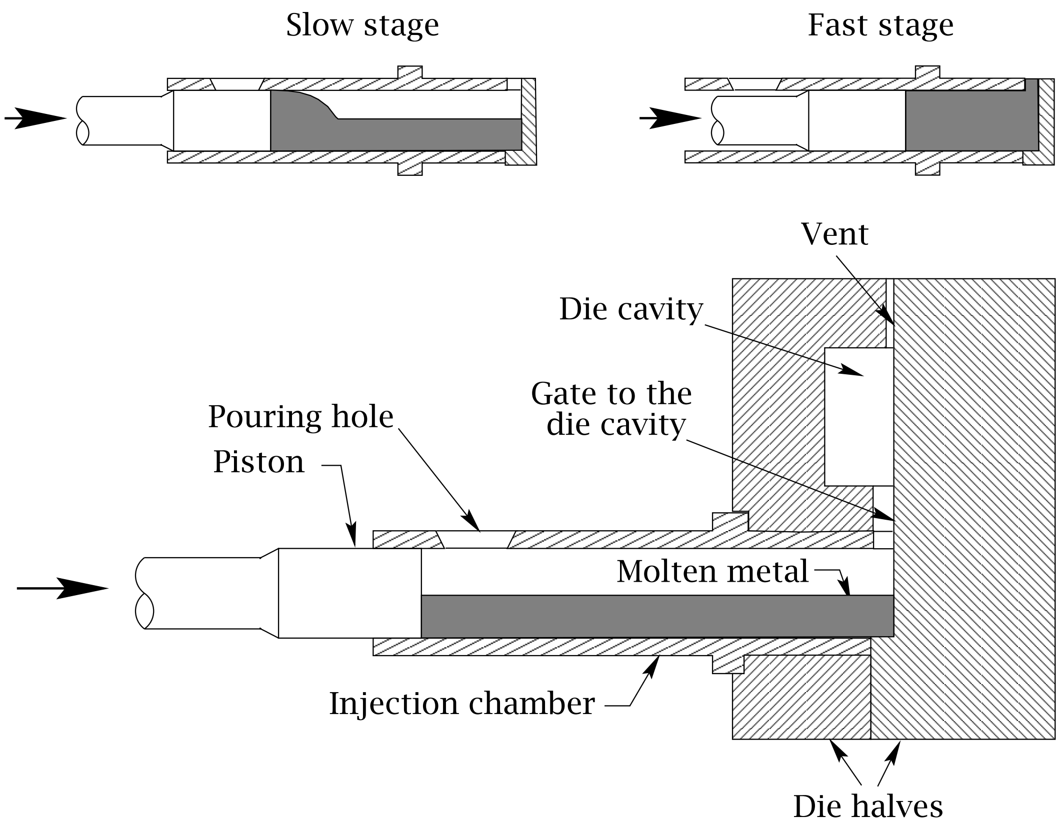
Talk outline

- Motivation
- Objectives
- Governing equations
- Numerical procedure
- Application of the model to breaking waves
  - Description of the problem
  - PHOENICS settings
  - Discussion of results
- Conclusions

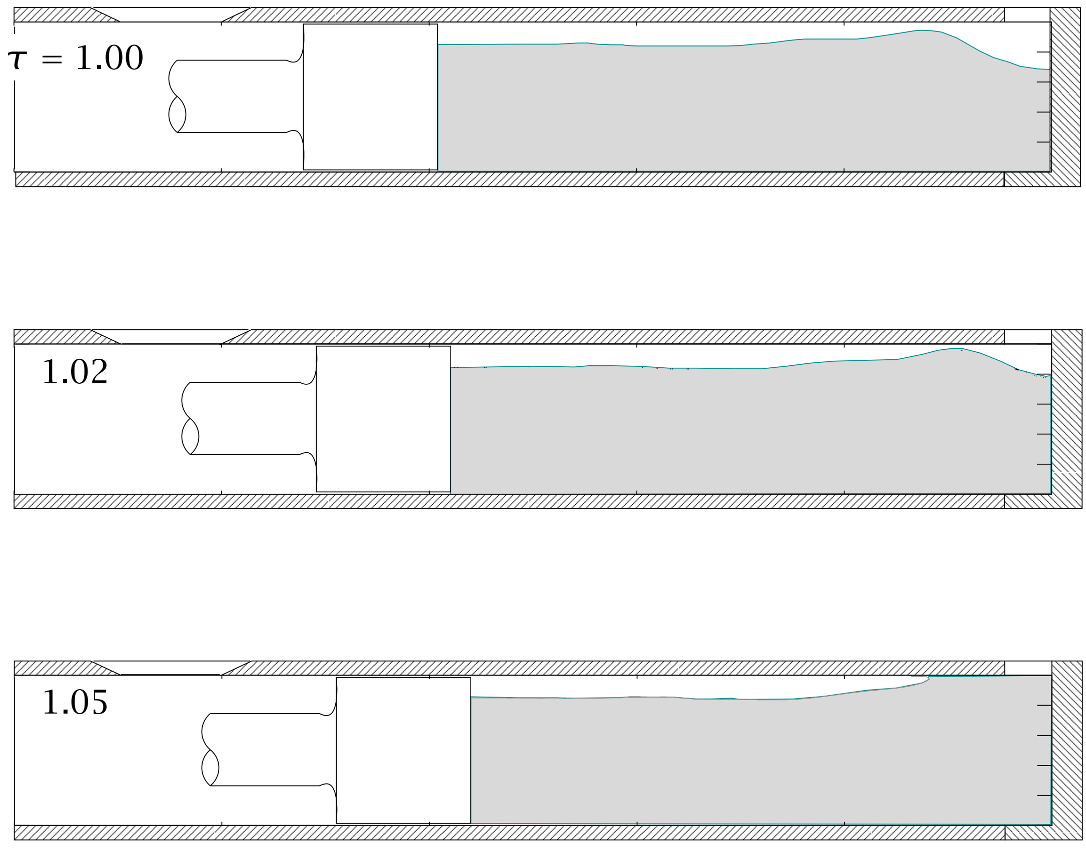
Motivation for work

- Die casting in horizontal cold chambers.
- Molten metal injected into a die cavity from a horizontal shot sleeve in which the metal is pushed by a plunger.
- Entrapment in the molten metal of initial air in the die cavity and in the shot sleeve ⇛ porosity when the metal solidifies.



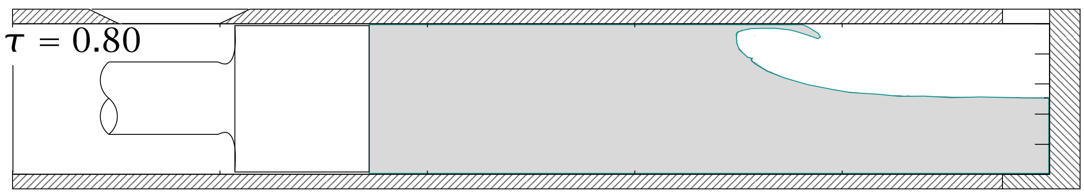
- Possible situations that increase air entrapment
  - Wave reflection against the end wall of the shot sleeve, caused by a too slow plunger motion

$$X'_{\max} = 0.7u_H$$

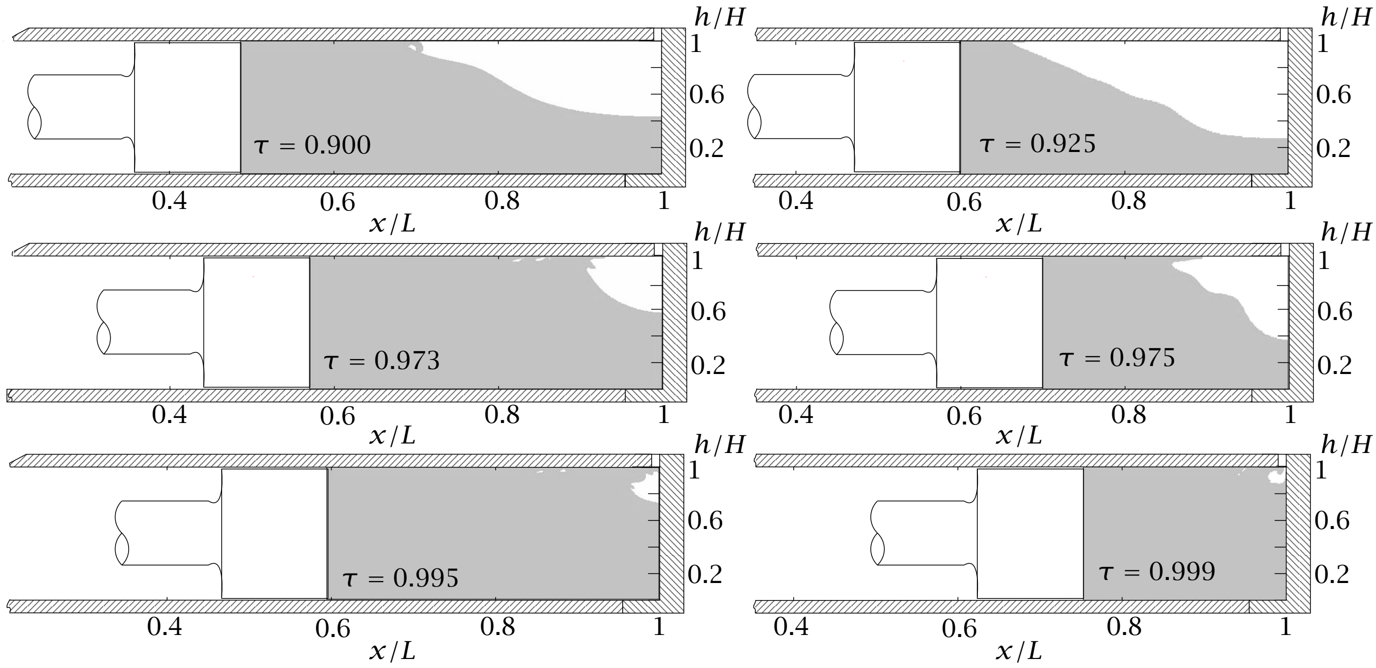


  - Roll over of the wave front caused by wave reflection against the shot sleeve ceiling or large plunger accelerations.

$$X'_{\max} = 1.5u_H$$



- Volume of trapped air in the shot sleeve mainly depends on
  - shot sleeve geometry
  - initial filling fraction
  - law of plunger motion



(a) (b)

Previous work

- Gómez et al. (2000) $\Rrightarrow$ simulation of the flow in the injection chamber using the Scalar Equation Method (SEM) implemented in PHOENICS.
- Relative motion between the piston and the end wall of the chamber $\Rrightarrow$ reproduced using a Cartesian grid, contracted by moving either the piston surface or the end wall (piston assumed to be at rest/horizontal body force per unit mass).
- Comparison with results obtained using a model based on the shallow-water approximation (also taking into account effects of wave reflection against the end wall of the chamber) (López et al., 2000a).
- Comparison with results obtained with a finite-element CFD code (Hernández et al., 1999; López et al., 2000a) specifically developed for free-surface flow modeling (Sant and Backer, 1995).
- For certain operating conditions, it was difficult to obtain grid-independent results, in particular after the beginning of wave breaking.

Previous work (continued)

- Numerical resolution of the scalar transport equation using continuum advection techniques $\Rrightarrow$ high levels of numerical diffusion.
- The scalar equation method implemented in PHOENICS tries to reduce the numerical diffusion by using the explicit discretization scheme of Van Leer (1977).
- Gómez et al. (2000) already mentioned that one of the purposes in future works would be to eliminate the unacceptable broadening of the region where the fluid marker variable F varies, which in many operating conditions was unavoidable even when the higher-order monotonic scheme of Van Leer (1977) or other similar techniques were used.

Objectives

- To implement in PHOENICS a numerical model for the simulation of interfacial and free-surface flows, based on a level set method.
- To apply the model to the simulation of the early stages of wave breaking in shallow-water, including the impact of the jet formed at the crest of the wave on the forward face of the wave, and the subsequent splash-up and impact cycles.
- In particular, we consider the waves generated by moving a piston in a two-dimensional injection chamber of a high-pressure die casting machine under operating conditions far from the optimal.
- To compare the results obtained with PHOENICS with those of a code specifically developed for the simulation of interfacial flows, whose efficiency and accuracy have been assessed by applying the code to solving some benchmark problems.

Governing equations

- The flow in both fluids is incompressible, two-dimensional and governed by the Navier-Stokes equations. Density and viscosity are uniform in each fluid.
- Surface tension, heat transfer and solidification effects are neglected.
- In the level set formulation (Osher and Sethian, 1988; Sussman et al., 1994, 1998; Sethian, 1999), the Navier-Stokes equations can be written as

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{\nabla p}{\rho(\phi)} + \frac{1}{\rho(\phi)} \nabla \cdot [2\mu(\phi)\mathcal{D}] + \boldsymbol{F},$$
$$\nabla \cdot \boldsymbol{u} = 0.$$

- At any time, the free surface is the zero level set of a higher-dimensional function ϕ: $\Gamma = \{\boldsymbol{x} | \phi(\boldsymbol{x}, t) = 0\}$. Given an initial level set function, $\phi(\boldsymbol{x}, t = 0)$, the evolution of ϕ is determined from

$$\partial\phi/\partial t + \boldsymbol{u} \cdot \nabla\phi = 0.$$

- Both the density and viscosity are determined from

$$\rho(\phi) = \rho_a + (\rho_l - \rho_a)H_\varepsilon(\phi), \quad \mu(\phi) = \mu_a + (\mu_l - \mu_a)H_\varepsilon(\phi),$$

where H_ε is a smoothed Heaviside function,

$$H_\varepsilon(\phi) = \begin{cases} 1 & \text{if } \phi > \varepsilon, \\ \frac{1}{2}[1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin(\frac{\pi\phi}{\varepsilon})] & \text{if } |\phi| \le \varepsilon, \\ 0 & \text{if } \phi < -\varepsilon. \end{cases}$$

- ε is a half of the computational interface thickness if ϕ is a distance function (signed normal distance to the free surface).

- If initially the level set function is a distance function, the solution to the ϕ equation does not necessarily remain a distance function $\Rrightarrow$ a procedure is needed to reinitialize it as a such without changing its zero level set $\Rrightarrow$ the interface thickness is maintained fixed in time.

- Two different methods have been considered in this work to reinitialize the level set function: the procedure proposed by Sussman et al. (1994) and the improved method proposed by Sussman et al. (1998).

- In the first method, given a level set function $\phi_0(\boldsymbol{x}, t)$, computed from Eq. (8) at time t, redistancing is carried out by solving the following equation to steady state:

$$\frac{\partial \phi}{\partial \tau} = S_\epsilon(\phi_0)(1 - |\nabla \phi|),$$

with the initial condition

$$\phi(\boldsymbol{x}, \tau = 0) = \phi_0(\boldsymbol{x}, t).$$

where $S_\epsilon(\phi_0) = 2(H_\epsilon(\phi_0) - 1/2)$, and τ is an artificial time.

- The level set function is first reinitialized near the interface (Sussman et al., 1998) $\Rrightarrow$ Since we need ϕ to be a distance function only in the vicinity of the interface, it is not necessary to solve the reinitialization equation to steady state over the whole domain.

- Sussman et al. (1999b) improved this method by introducing a constraint based on volume conservation,

$$\partial_\tau \int_{\Omega_{i,j}} H_\varepsilon(\phi)\,\mathrm{d}x\,\mathrm{d}z = 0.$$

- Reinitialization equation is modified and written as (Sussman et al., 1999b)

$$\frac{\partial \phi}{\partial \tau} = S_\epsilon(\phi_0)(1 - |\nabla\phi|) + \lambda_{i,j} f(\phi_0) = \mathcal{L}(\phi_0, \phi) + \lambda_{i,j} f(\phi_0),$$

which must be solved with the initial condition

$$\phi(\boldsymbol{x}, 0) = \phi_0(\boldsymbol{x}, t),$$

and where

$$f(\phi_0) = H'_\varepsilon(\phi_0)|\nabla\phi_0| \approx H'_\varepsilon(\phi_0)$$

($H'_\varepsilon(\phi) = \partial H_\varepsilon(\phi)/\partial\phi$), which ensures that the correction is applied only at the interface. $\lambda_{i,j}$, which is assumed constant in each cell, is then obtained from

$$\partial_\tau \int_{\Omega_{i,j}} H_\varepsilon(\phi)\,\mathrm{d}x\,\mathrm{d}z = \int_{\Omega_{i,j}} H'_\varepsilon(\phi)\frac{\partial\phi}{\partial\tau}\,\mathrm{d}x\,\mathrm{d}z \simeq \int_{\Omega_{i,j}} H'_\varepsilon(\phi_0)\frac{\partial\phi}{\partial\tau}\,\mathrm{d}x\,\mathrm{d}z$$
$$= \int_{\Omega_{i,j}} H'_\varepsilon(\phi_0)[\mathcal{L}(\phi_0,\phi) + \lambda_{i,j} f(\phi_0)]\,\mathrm{d}x\,\mathrm{d}z = 0,$$

which gives

$$\lambda_{i,j} = \frac{-\int_{\Omega_{i,j}} H'_\varepsilon(\phi_0)\mathcal{L}(\phi_0,\phi)\,\mathrm{d}x\,\mathrm{d}z}{\int_{\Omega_{i,j}} H'_\varepsilon(\phi_0) f(\phi_0)\,\mathrm{d}x\,\mathrm{d}z}.$$

Numerical procedure

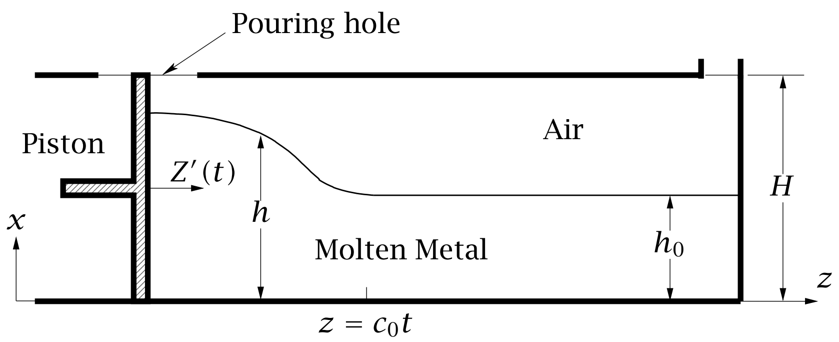
- Two different discretization schemes of the convective term in the momentum equation, hybrid and MUSCL (see, for example, Waterson and Deconinck, 1995), were considered.
- The continuity equation is solved in PHOENICS as the basis of the GALA algorithm in terms of volumetric conservation, avoiding the need of evaluating an average density.
- Transport equation of ϕ was solved using a method similar to the predictor-corrector methods used by Puckett et al. (1997), Almgren et al. (1998) and Sussman et al. (1999a) to discretize the convective term. The method is based on the unsplit Godunov method introduced by Colella (1990).
- Reinitialization equation was solved using a second-order Runge-Kutta scheme for discretization in time and a second-order Essentially Non-Oscillatory (ENO) scheme for spatial discretization (Shu and Osher, 1989).
- In equation for λ, $\mathcal{L}(\phi_0, \phi)$ is estimated as an approximation to $\partial\phi/\partial\tau$: $(\tilde{\phi}^{k+1} - \phi_0)/\tau^{k+1}$, where $\tilde{\phi}^{k+1}$ is the value obtained from the reinitialization eq. at the false time step $k+1$. The distribution of ϕ^{k+1} at the end of time step $k+1$ is obtained from

$$\phi^{k+1} = \tilde{\phi}^{k+1} + \tau^{k+1}\lambda_{i,j}H'_\epsilon(\phi_0).$$

- A detailed description of the numerical procedure can be found in Gómez (2002).

Application of the model to breaking waves

- Previous work (Hernández et al., 2001) ⇛ results for the evolution of water waves in shallow water until the instant at which the jet formed at the wave crest impacts onto the forward face of the wave.
- In this paper ⇛ results for the evolution of breaking waves, such as those generated in a die casting injection chamber, after the re-entry of the first plunging jet onto the wave's forward face and during subsequent splash-up cycles.



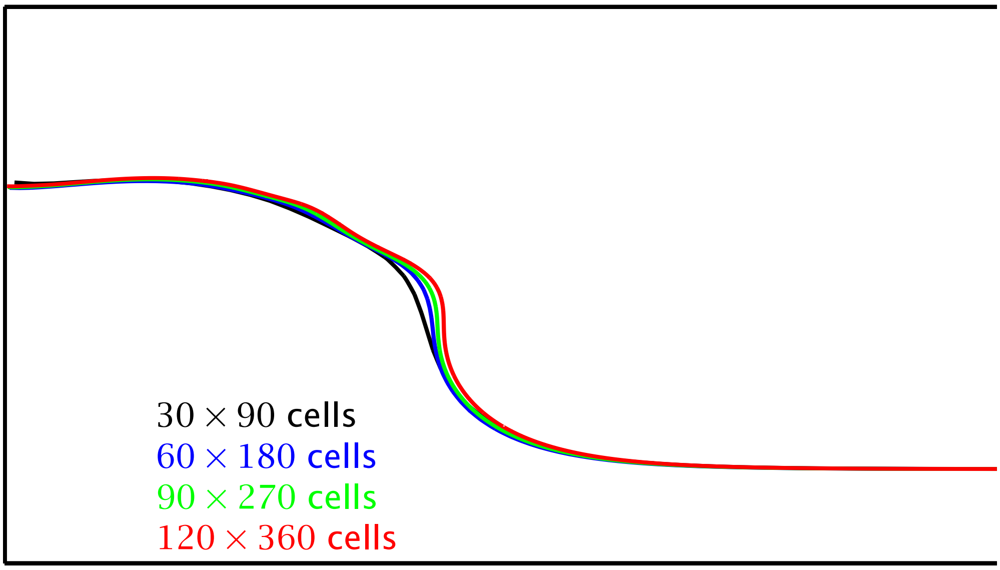
- Fluids are initially at rest. Piston motion laws:

$$Z''(t) = \alpha^2 \beta e^{\alpha t}, \quad Z''(t) = \frac{2}{3}\frac{c_0^2}{\ell}\left(1 - \frac{c_0}{\ell}t\right)^{-4/3}, \quad \text{for } t \le t_H.$$

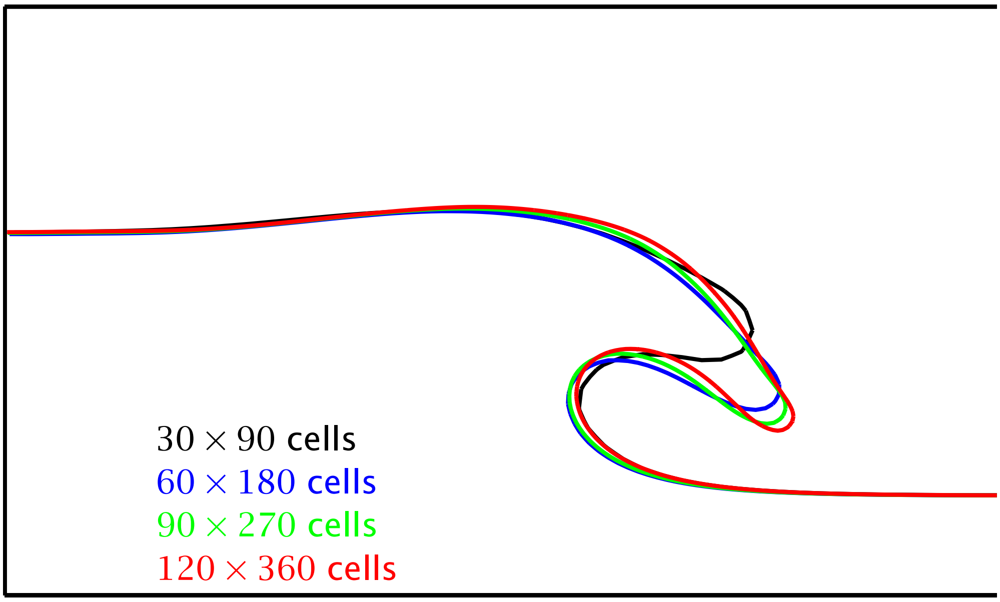
- t_H is sufficiently lower than the time at which the molten metal reaches the chamber ceiling (fictitious chamber of height $H' = 1.5H$).
- Fixed computational mesh. Effects of piston motion reproduced by considering a body force per unit mass acting on the molten metal.
- Effects of the relative motion between the ceiling and bottom walls of the injection chamber and the piston were neglected. A slip boundary condition specified at the chamber walls. Pressure fixed at the gate to the mold cavity.

PHOENICS settings

- The implementation of the level set method was carried out in a specific subroutine.
- In all the simulations, the thickness of the interface (2ε) was taken to be equal to twice the cell size, and the CFL number (based on the maximum velocity in the domain) was set equal to 0.05.
- We needed to reinitialize ϕ every time step in order to keep it close to a signed distance function.
- The false time step in the resolution of the reinitialization eq. was set equal to $\Delta x/2 = \epsilon/2$, a value which makes CFL ≤ 0.5 because $|S_\epsilon(\phi_0)\frac{\nabla\phi}{|\nabla\phi|}| \leq 1$ (reinit. eq. can be written as $\partial\phi/\partial\tau + [S_\epsilon(\phi_0)\nabla\phi/|\nabla\phi|] \cdot \nabla\phi = S_\epsilon(\phi_0)$).
- The resolution of the reinitialization eq. was stopped after three false time steps, when the reinitialization reaches a distance from the interface of about one and a half grid cells.
- A uniform grid of size 60×180 cells in a subdomain of size $H' \times L/2$ (where the wave breaking process occurs in all the cases presented below) has been used in most of the simulations.

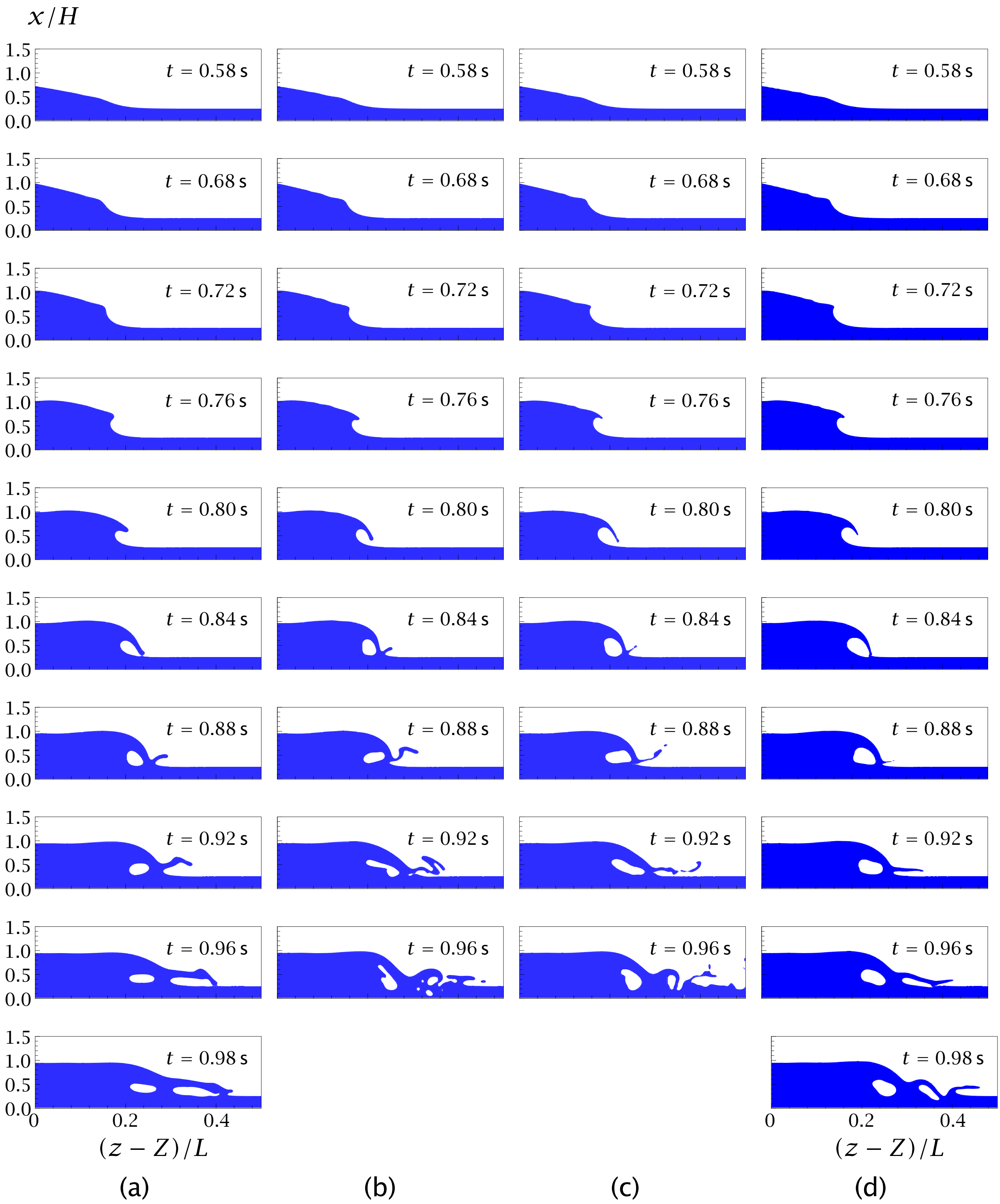


(a)

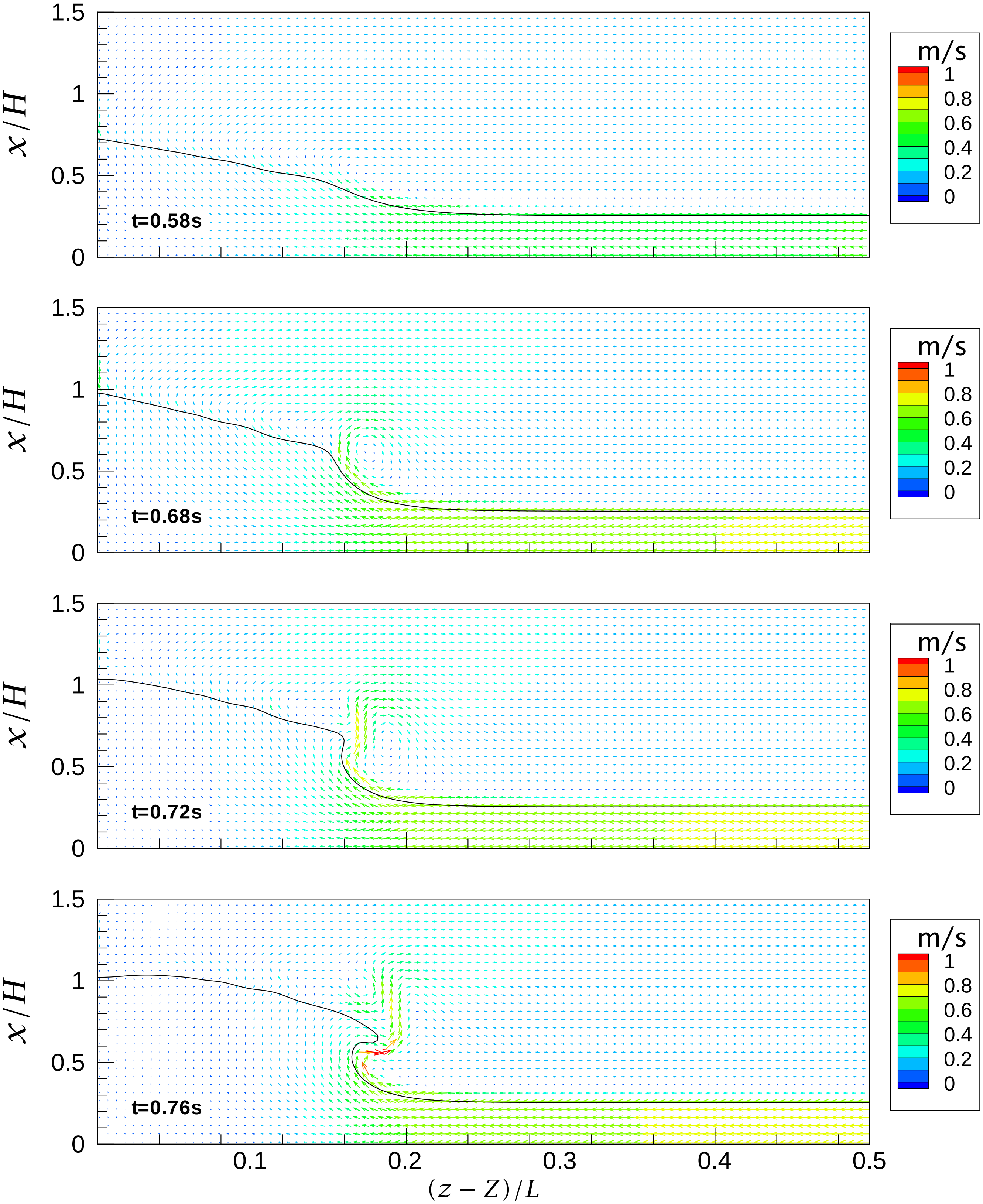


(b)

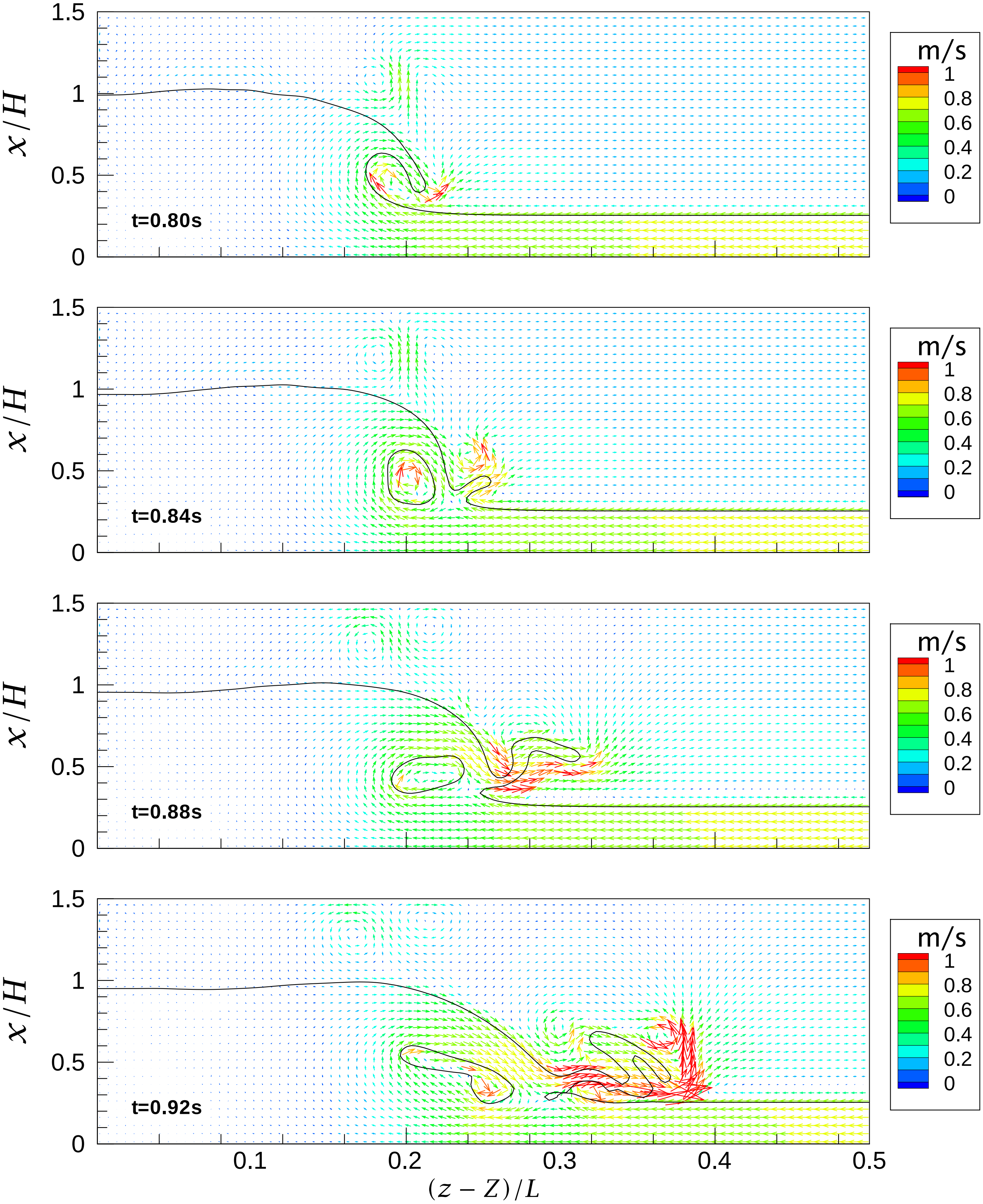
Dependence of the wave profiles obtained with PHOENICS on the grid size in a subdomain of size $H' \times L/2$. Second piston acceleration law, with $\ell = 0.45L$ and $t_H = 0.5$ s. a) $t = 0.6$ s; b) $t = 0.7$ s.

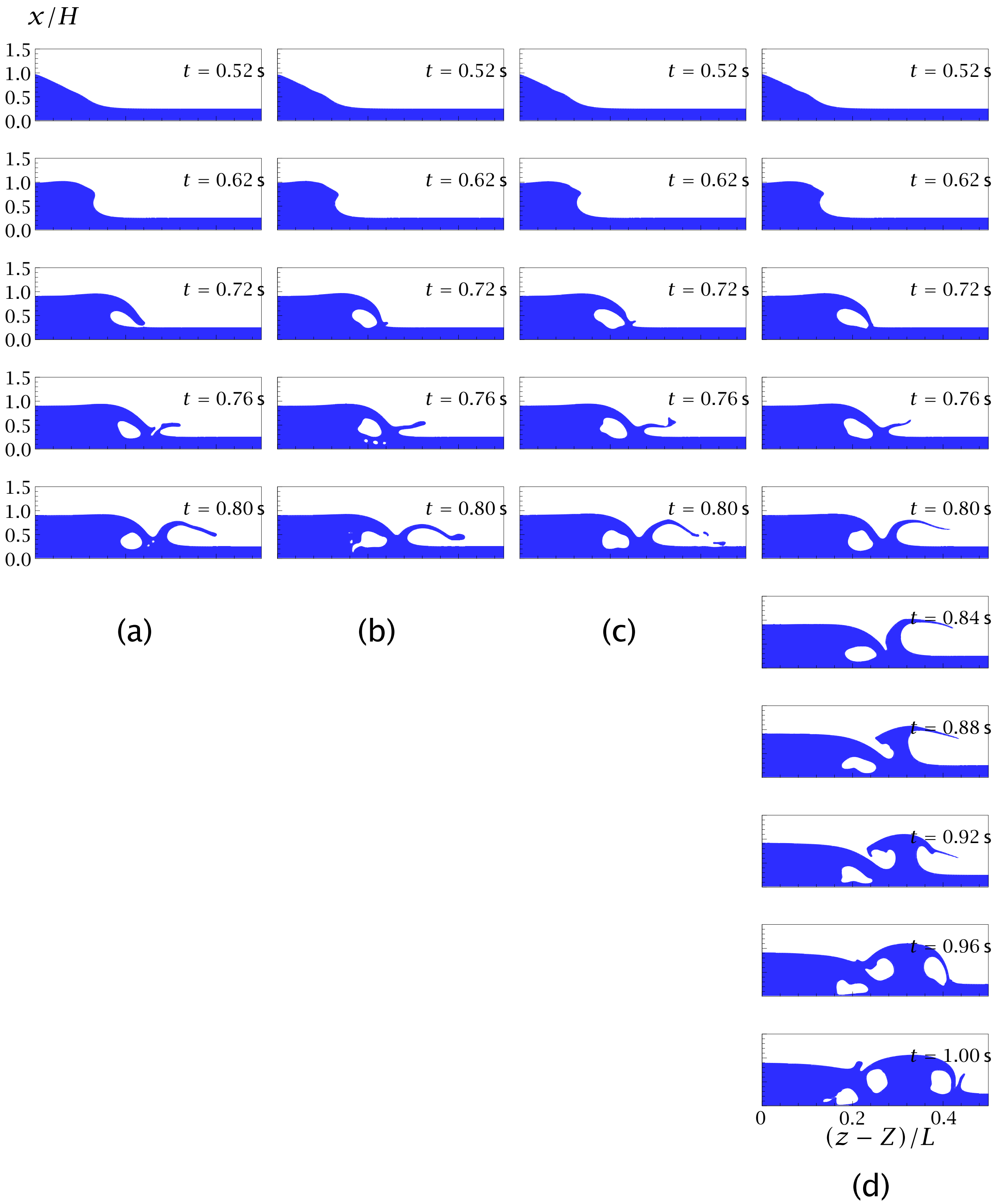


Wave profiles obtained for $H = 5.08$ cm, $f = 0.254$, and the first piston acceleration law with $\xi = 2$ and $z_c = 0.5L$. a) PHOENICS; hybrid scheme. b) PHOENICS; MUSCL scheme. c) Our code; reinitialization method of Sussman et al. (1998). d) Our code; method of Sussman et al. (1994) with local grid refinement.



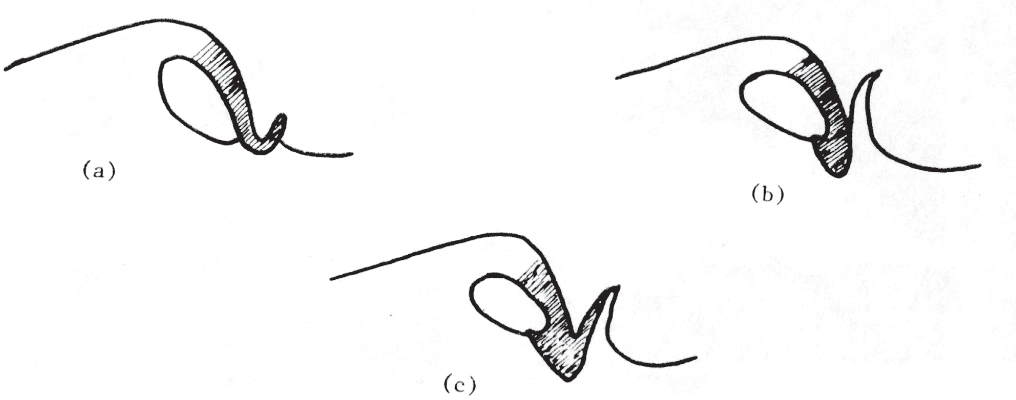
Velocity vector distributions obtained with PHOENICS (MUSCL scheme) for $H = 5.08$ cm, $f = 0.254$, and the first piston acceleration law with $\xi = 2$ and $z_c = 0.5L$.


t=0.80s
t=0.84s
t=0.88s
t=0.92s
x/H
(z − Z)/L
m/s
1
0.8
0.6
0.4
0.2
0
1.5
0.5
0.1
0.2
0.3
0.4
0.5

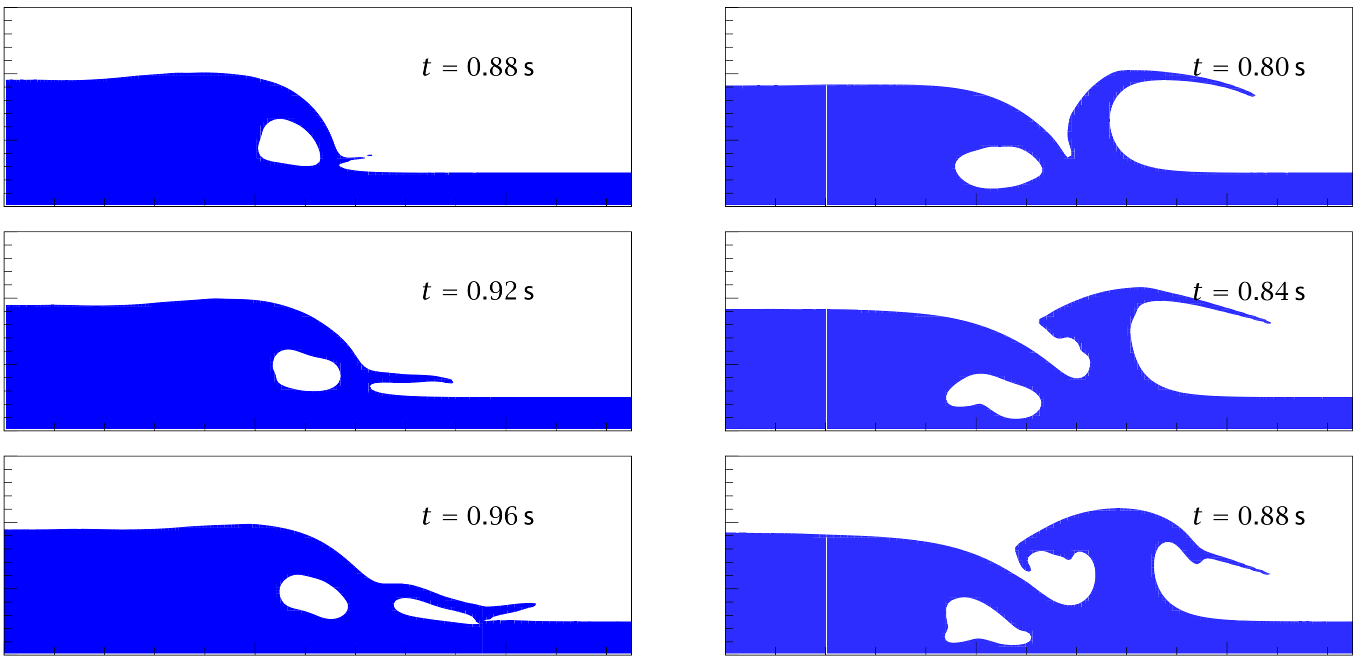


Wave profiles obtained for $H = 5.08$ cm, $f = 0.254$, and the second piston acceleration law with $\ell = 0.45L$ and $t_H = 0.5$ s. a) PHOENICS; hybrid scheme. b) PHOENICS; MUSCL scheme. c) Our code; reinitialization method of Sussman et al. (1998). d) Our code; method of Sussman et al. (1994) with local grid refinement.

Possible mechanisms of splash up



(Peregrine, 1983)



Conclusions

- The model predicts well the overall characteristics of the flow, and the results show the capabilities and limitations of the model in simulating the wave breaking problem with accuracy and efficiency.
- The results obtained with PHOENICS have been compared with those of a code specifically developed for the simulation of interfacial flows, which is based on a finite-difference discretization of the Navier-Stokes equations, a second-order approximate projection method and a level set method similar to that implemented in PHOENICS, and whose efficiency and accuracy have been assessed by applying the code to solving some benchmark problems.
- The accuracy in the description of interfaces and the range of applicability of the present model clearly excel those of other models previously implemented in PHOENICS, such as the Scalar Equation Method or the Height of Liquid Method.

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