Numerical Computation of Multi-phase Flows
A Lecture Course

by

D Brian Spalding

November 1981 HTS/81/8

Presented at Imperial College

2-6 November 1981

Computational Fluid Dynamics Unit
Room 440 ME Building
Imperial College of Science and Technology
Exhibition Road
London
UK

NUMERICAL COMPUTATION OF MULTI-PHASE FLOWS

A Lecture Course

by

D. Brian Spalding

Imperial College, London SW7.

November 1981

C O N T E N T S

P R E F A C E

The territory of computational fluid mechanics has recently been much enlarged by its annexation of two-phase-flow phenomena. This enlargement has resulted, in part, from the necessity to predict realistically the accident-situation flows which may arise in nuclear power plants cooled by pressurised water; for breaks in vessels or pipe-lines can give rise to the formation of steam; and how the steam and water flow out, and how the injected emergency coolant flows in, affect crucially whether the accident will develop into a disaster.

The main characteristics of such flows, and the ones which create the special difficulties which the numerical analyst must overcome, are:-

- Two fluids are dispersed within, and so "share", the same space; each thus denies to the other the volume which it occupies itself.
- These fluids engage in frictional, thermal and mass-transfer interactions with each other, at rates which depend upon the local relative velocity, temperature difference etc.
- There are twice the usual number of equations to solve for momentum and for temperature; and the volume fractions of the two phases must also be computed at each point within the field.

The increase in the number of equations affects the difficulty of solution qualitatively as well as quantitatively; for it invalidates solution algorithms which are satisfactory for single-phase flows. Indeed, so troublesome did the augmented interconnectedness of the mathematical problem first prove to be that, for a time, it was thought to be inherently intractable; and many publications during the period 1974-1979 discussed the so-called "ill-posedness" of the problem presented by the governing differential equations. Experience proved, however, that the equations do present "well-posed" problems: the solutions do exist, and can be found numerically; the task is just that of devising a reliable means of finding out what they are.

The present lectures are largely concerned with a particular means, the so-called IPSA algorithm.*

---

*IPSA stands for interphase-slip algorithm.

Contd...

It has been used by the present writer and his colleagues since 1976; and it has proved to be satisfactory for use in all the situations to which it has been applied. These include both steady and unsteady flows, whether in one, two or three space dimensions; and the presence or absence of heat transfer, mass transfer and chemical reaction appear not to make any difference to its reliability.

The lectures were first presented at a Course held at Purdue University in the Spring of 1978, which was when the two-phase computer code GENMIX 2P first saw the light of day. Since that time, they have been altered in detail, and augmented; but their main structure has not appeared to necessitate change.

The phenomena which IPSA can help to predict are very numerous; and they are by no means confined to the nuclear-power industry. For example, the combustion of coal in a pulverized-fuel furnace, or in a fluidized bed, is a two-phase phenomenon, in which the two phases (i.e. gas and coal particles) slip relative to one another. IPSA has been used for the computer modelling of the whole process, with allowance for effects of radiation and of chemical reactions.

Of the same essential kind are the phenomena which occur in a gun barrel, when an inflammation wave passes through the compressed bed of solid-propellant pellets, causing the evolution of hot gases which raise the pressure, set the projectile in motion, and then flow through the bed in pursuit. Friction between the gas and the particles sets the latter in motion also; so the result is a two-phase flow with slip. Here, too, IPSA has been helpful in procuring solutions to the relevant partial differential equations, and so in aiding those who wish to understand quantitatively what happens.

For a final example of how IPSA has been used, the flow of natural gas in under-ocean pipelines can be mentioned. In these pipelines, there is often a flow of liquid condensate also; and, because the diameters of the pipes are large, this liquid usually forms a "stratified" layer at the bottom of the pipe. Stratification of this kind can give rise to gravity-wave processes, some of which have important (and even damaging) consequences. The stratification option also appears in IPSA; and it will be discussed in the present lecture series.

Computer codes which will predict phenomena of the kind which have just been described are necessarily large and complex; and prospective users require extensive documentation, and prolonged periods of instruction, before their use can be successful. It is far from being possible to provide such codes, documentation, or instruction within the framework of the present lecture series.

Nevertheless, in order that participants in the lecture course should attain some insight into how such codes work, and some practical experience of using them, a listing is provided of the GENMIX 2P computer program; and lectures 4 and 8 provide an introduction to the use of this program.

Full documentation for GENMIX 2P is <u>not</u> provided; but it is also not needed; for GENMIX 2P is simply a two-phase version of a program which has been available to the public for many years, and about which a full written description has been published.

Numerous colleagues, students and assistants deserve credit for their contributions to the present work. Some have contributed by using earlier versions of IPSA, and experiencing the results of their shortcomings; others have helped me, by questions or comments, to clarify either the basic ideas or their implementation in detail; others yet again have assisted in the production of the present document. Since the names of those who have made technical contributions will appear at various places in the reference lists, or elsewhere in the lectures, I here express my thanks only to those in the last-named category, namely Ms. Colleen King, who typed the first version, and Miss Susan Farmiloe, Mrs. Maggie Dean and Mrs. Frith Oliver, who have successively improved or augmented it.

Improvements and augmentations will certainly be needed in the future. Suggestions will be greatly welcomed.

<u>D. Brian Spalding</u>

REFERENCES

Amsden A.A. and Harlow F.H.
"The SMAC Method"
Los Alamos Scientific Laboratory Report No. LA-4370, 1970

Amsden A.A. & Harlow F.H.
"A Simplified MAC Technique for Incompressible Fluid Flow Calculations"
Los Alamos Report LA-DC-11272, 1970

Baghdadi A.H.A.
"Numerical Modelling of Two-Phase Flow with Interphase Slip"
PhD Thesis, Imperial College 1979

Baghdadi A.H.A., Rosten H.I., Singhal A.K., Spalding D.B., Tatchell D.G.
"Finite-Difference Predictions of Waves in Stratified Gas-Liquid Flows"
In Durst F., Tsiklauri G.V., Afgan N.H. "Two-Phase Momentum, Heat and Mass Transfer" Hemisphere Publishing Corporation, Washington D.C. 1979

Banerjee S. & Hancox W.T.
"Transient Thermohydraulics Analysis for Nuclear Reactors"
Sixth International Heat Transfer Conference Proceedings
Vol. 6 pp 331-338, 1978

Banerjee S. and Chan A.M.C.
"Models for Transient Two-phase Flows"
in "Basic Two-phase Flow Modeling in Reactor Safety and Performance" EPRI Conference, Tampa, Florida, March 2 1979

Butterworth D. & Hewitt G.F. (Editors)
"Two-Phase Flow and Heat Transfer
Oxford University Press, Oxford, 1977

Caretto L.S., Gosman A.D., Patankar S.V. & Spalding D.B.
"Two Calculation Procedures for Steady, Three-Dimensional Flows with Recirculation"
Proceedings of the 3rd International Conference on Numerical Methods in Fluid Mechanics
Vol II, pp 60-68 1973 Springer-Verlag Heidelberg, Editors: Ehlers J., Hepp K. & Weidenmüller H.A.

Collier J.G.
"Convective Boiling and Condensation"
McGraw Hill, London 1972

Crowe C.T. & Choi H.N.
"Two-dimensional numerical Model for Steam-water Flow in a Sudden Contraction"
In Two-Phase Momentum Heat and Mass Transfer, Editors Durst F., Tsiklauri G.V. & Afgan N., Hemisphere, Washington 1979

Dallman J.C. & Hanratty T.J.
"Interpretation of Entrainment Measurements in Annular Gas-Liquid Flows"
In Durst F., Tsiklauri G.V. & Afgan N.H. Two-Phase Momentum, Heat and Mass Transfer. Hemisphere Publishing Corporation, Washington D.C. 1979.

Delhaye J.M.
"Space-averaged Equations and Two-Phase Modeling"
In:Durst F., Tsiklauri G.V., Afgan N.H. "Two-Phase Momentum Heat and Mass Transfer", Hemisphere, Washington 1979

Durst F., Tsiklauri G.V., Afgan N.H.
"Two-Phase Momentum, Heat and Mass Transfer"
Hemisphere Publishing Corporation, Washington D.C., 1979

Elghobashi S.E., Pratt D.T., Saplding D.B. & Srivatsa S.K.
"Unsteady Combustion of Fuel Spray in Jet-Engine Afterburners"
3rd Air-Breathing Engines Conference, Munich, 1976

Fisher S.A., Pearce D.L.
"A Theoretical Model for Describing Horizontal annular Flows"
In Durst F., Tsiklauri G.V. & Afgan N.H. Two-Phase Momentum, Heat and Mass Transfer. Hemisphere Publishing Corporation, Washington D.C. 1979

Gibson M.M. & Morgan B.B.
"Mathematical Model of Combustion of Solid Particles in a Turbulent Stream with Recirculation"
Journal of the Institute of Fuel, Vol 43, pp 517-523 1970

Gidaspow, D.
"Modeling of Two-phase Flow"
Round Table Discussion (RT 1-2) Proceedings 5th International Heat Transfer Conference 1974 Vol VII p 163

Harlow F.H.
"Comments at Round Table Discussion"
(RT 1-2) Proceedings 5th International Heat Transfer Conference 1974 Vol VII p 164

Harlow F.H. and Amsden A.A.
"Multi-fluid flow Calculations at all Mach Numbers"
Journal Computational Physics Vol 16 p.1 1974

Harlow F.H. & Amsden A.A.
"Numerical Calculation of Multi-Phase Fluid Flow"
Journal Computational Physics, Vol 17 No. 1 pp19-52 1975

Harlow F.H. and Amsden A.A.
"Numerical Calculation of Almost Incompressible Flow"
Journal of Computational Physics Vol 3, 1 1968

Harlow F.H. & Welch J.E.
"Numerical Calculation of Time-dependent Viscous Incompressible Flow of Fluid with Free Surface"
Physics of Fluids Vol 8 No. 12 pp 2182-2189 1965

Hewitt G.F. & Hall-Taylor N.S.
"Annular Two-Phase Flow"
Pergamon Press, Oxford, 1970

Hulme G., Phelps P.J., Spalding D.B., Tatchell D.G.
"Prediction of Steady, Three-dimensional Flow in Pressurized-water Steam Generators"
In Boiler Dynamics and Control in Nuclear Power Stations, BNES, London 1979

Ishii M.
"Thermo-Fluid Dynamic Theory in Two-Phase Flow"
Reyrolles, Paris, 1975

Liles D.R. & Mahaffy J.H.
"Modeling of Large Pressurised Water Reactors"
in "Basic Two-phase flow modelling in Reactor Safety and Performance" EPRI Conference, Tampa, Florida, March 2 1979

Kurosaki Y. & Spalding D.B.
"One-Dimensional Unsteady Two-Phase Flows with Interphase Slip: A Numerical Study"
2nd Multi-Phase Flow and Heat Transfer Symposium, Miami 1979

Lahey R.T. and Drew D.A.
"The Analysis of Phase Distribution in Fully-Developed Two-Phase Flows"
2nd Multi-Phase Flow and Heat Transfer Symposium, Miami Beach, April 1979

Launder B.E., Spalding D.B.
"Mathematical Models of Turbulence"
Academic Press, London & New York, 1972

Marchand E.O., Singhal A.K. & Spalding D.B.
"Predictions of Operation Transients for a Steam Generator of a PWR Nuclear Power System"
Paper for presentation at 1980 ASME Nuclear Division Conference August 18-21 1980

Markatos N.C., Moult A. Phelps P.J. & Spalding D.B.
"The Calculation of Steady Three-Dimensional Two-Phase Flow and Heat Transfer in Steam Generators"
In Durst F., Tsiklauri G.V. & Afgan N.H. Two-Phase Momentum, Heat and Mass Transfer. Hemisphere Publishing Corporation, Washington D.C. 1979

Maxwell T.T. & Spalding D.B.
"Numerical Modelling of Free-Surface Flows"
Paper for Pressure Vessels and Piping Conference, San Francisco, August 12-15 1980

McCoy D.D., Hanratty T.J.
"Droplet Mixing and Deposition in Turbulent Flow"
In Durst F., Tsiklauri G.V., Afgan N.H. Two-Phase Momentum, Heat and Mass Transfer. Hemisphere Publishing Corporation, Washington D.C. 1979

Michiyoshi I.
"Two-Phase Two-Component Heat Transfer"
Sixth International Heat Transfer Conference Proceedings
Vol 6, pp 219-233, 1978

Mongia H.G., Smith K.
"An Empirical/Analytical Design Methodology for Gas-Turbine Combustors"
AIAA Paper Number 78-998 1978

Moult A., Pratap V.S., Spalding D.B.
"Calculation of Unsteady One-Dimensional Two-Phase Flows"
First International Conference on PhysicoChemical Hydrodynamics, Oxford, 1977 Editor: Spalding D.B., Published by Advance, Channel Islands, 1978

Patankar, S.V.
"Numerical Heat Transfer and Fluid Flow"
Hemisphere Publishing Corporation 1980

Patankar S.V., Spalding D.B.
"A Calculation Procedure for Heat, Mass and Momentum Transfer in Three-Dimensional Parabolic Flows"
International Journal of Heat and Mass Transfer, Vol 15, pp 1787-1806, 1972

Patankar S.V., Spalding D.B.
"A Calculation Procedure for the Transient and Steady-State Behaviour of Shell-and-Tube Heat Exchangers"
Heat Exchanger Design and Theory Sourcebook, Scripta, Washington D.C., 1976

Patankar S.V., Spalding D.B.
"Computer Analysis of the Three-Dimensional Flow and Heat Transfer in a Steam Generator"
Forschung Ing-Wes, Vol 44, pp 47-52, 1978

Pratap V.S. & Spalding D.B.
"Numerical Computations of the Flow in Curved Ducts"
Aero Quarterly, Vol 26, pp 219-228, August 1975

Pun, W.M., Spalding D.B., Rosten H., Svensson U.
"Calculation of 2D Steady Two-Phase Flows"
Proceedings ICHMT 1978: Two-Phase Momentum, Heat & Mass Transfer, Editors: Durst, Tsiklauri, Afgan, Hemisphere 1979, Vol 1, pp 461-470, 1979.

Ramshaw J.D. and Trapp J.A.
"A Numerical Treatment for Low-speed Homogeneous Two-Phase Flow with Sharp Interfaces"
Journal Computational Physics, Vol 21 1976 pp 438-453

Sala R., Spalding D.B.
"A Mathematical Model for an Axi-Symmetrical Diffusion Flame in a Furnace"
La Rivista dei Combustibili, Vol XXVII, pp 180-186. 1973

Singhal A.K. & Spalding D.B.
"Numerical Modelling of Two-Phase Flow in Steam Generators"
Second Multi-Phase Flow and Heat Transfer
Symposium Miami Beach, April 1979

Singhal A.K., Spalding D.B. & Marchand, E.O.
"Parametric Computations for Thermal-Hydraulic Performance of a PWR Steam Generator"
1980 ASME Nuclear Division Conference, Saratoga, New York, October 6-8, 1980

Singhal A.K., Keeton L.W. & Spalding D.B.
"Predictions of Thermal Hydraulics of a PWR Steam Generator by Using the Homogeneous and Two Slip-Flow Models"
Paper submitted to ASME 19th National Heat Transfer Conference, Orlando, Florida, July 27-30 1980

Spalding D.B.
"Mathematical Modelling of Fluid-mechanics, Heat-transfer and Mass-transfer processes"
Imperial College, London, Heat transfer Section Report ref. HTS/80/1.

Spalding D.B.
"Convective Mass Transfer"
Edward Arnold, London, 1963

Spalding D.B.
"One Dimensional Two-Phase Flow, with Unequal Velocities"
Imperial College London, Heat Transfer Section Report ref. HTS/79/10, 1979

Spalding D.B.
"Notes on the PARMIX Program"
Imperial College, London, Heat Transfer Section Report, ref: BL/TN/A/35, 1970

Spalding D.B.
"Numerical Computation of Multi-Phase Fluid Flow and Heat Transfer"
Contribution to "Recent Advances in Numerical Mechanics"
Editor C. Taylor, Pineridge Press, 1980

Spalding D.B.
"Numerical Computation of Two-Phase Flow"
Second International Conference on PhysicoChemical Hydrodynamics, Washington D.C. 1978

Spalding D.B.
"The Calculation of Free-Convection Phenomenon in Gas-liquid Mixtures"
ICHMT Seminar, Dubrovnik 1976. Published in "Turbulent buoyant convection" by Hemisphere Washington 1977 Editors N. Afgan and D.B. Spalding, pp 569-586

Spalding D.B.
"Multi-Phase Flow Predictions in Power-System Equipment and Components"
EPRI Workshop on Basic 2-phase Modelling in Reactor Safety and Performance Int J Multi-Phase Flow Vol 6 pp 157-168 1980

Soo S.L.
"Fluid Dynamics of Multi-Phase Systems"
Blaisdell, Waltham, 1967

Stuhmiller J.H.
"The Influence of Interfacial Pressure Forces on the Character of Two-Phase Flow Model Equations"
International Journal of Multiphase Flow, Vol 3, pp 551-600, 1977

Swithenbank J., Turan A., Felton P.G.
"Three-Dimensional Two-Phase Mathematical Modelling of Gas-Turbine Combustors"
Project SQUID Workshop on Gas-Turbine Combustor Design Problems, 1978

Wallis G.B.
"One-Dimensional Two-Phase Flow"
McGraw Hill, New York, 1969

MPF 1	1/30	NUMERICAL COMPUTATION OF MULTI-PHASE FLOWS BY D. BRIAN SPALDING LECTURE 1: INTRODUCTION AND OVERVIEW

TOPICS DISCUSSED:

- PRACTICAL APPLICATIONS:
  - LIQUID ~ SOLID;
  - GAS ~ SOLID;
  - GAS ~ SOLID;
- PHYSICS:
  - FRICTION;
  - HEAT TRANSFER;
  - MASS TRANSFER;
  - MECHANICAL EFFECTS;
- MATHEMATICS:
  - DIFFERENTIAL EQUATIONS;
  - FINITE-DOMAIN EQUATIONS;
  - SOLUTION PROCEDURE.
- EXAMPLES.
- CONCLUSIONS.

MPF 1	2/30	PRACTICAL APPLICATIONS: LIQUID ~ SOLID

<u>ENGINEERING</u>

- TRANSPORT OF COAL AS A SLURRY
- CLARIFICATION BY SETTLING
- FILTRATION
- CRYSTALLIZATION
- DEPOSITION IN PIPE LINES, BOILERS, REACTORS.

<u>NATURAL ENVIROMENT</u>

- EROSION OF RIVER BEDS
- SILTING OF ESTUARIES
- TURBIDITY CURRENTS (Under-water avalanches)
- GROUND-WATER MOVEMENT
- OIL~WATER FLOW IN POROUS ROCKS
- LANDSLIDES

MPF 1	3/30	PRACTICAL APPLICATIONS: GAS ~ SOLID; ENGINEERING.

<u>CHEMICALLY INERT</u>

- SEPARATION OF DUST FROM AIR IN A CYCLONE
- SAMPLING OF DUST-LADEN GASES
- PREVENTION OF SAND ENTRY TO HELICOPTER ENGINES
- FLUIDISED-BED HEAT-TRANSFER DEVICES
- SAND-BLAST-CLEANING PROCESSES

<u>CHEMICALLY REACTIVE</u>

- FLUIDISED-BED COAL COMBUSTORS
- COMBUSTION OF PROPELLANT PELLETS IN GUN BARRELS
- PRODUCTION OF CARBON BLACK OR TITANIUM OXIDE
- BURN-OUT OF SOOT IN OIL AND GAS FLAMES
- EXPLOSIONS OF COMBUSTIBLE DUSTS

MPF 1	4/30	PRACTICAL APPLICATIONS: GAS ~ SOLID; THE NATURAL ENVIRONMENT

<u>SAND AND AIR</u>

- DUNE FORMATION
- SAND-STORM PHENOMENA
- LIFTING AND TRANSPORT OF SOLID MATERIALS BY TORNADOES
- EROSION BY WIND ACTION
- "SALTATION"

<u>ICE AND AIR</u>

- FORMATION AND PRECIPITATION OF SNOW, AND OF HAIL
- AVALANCHE FORMATION AND BEHAVIOUR
- FORMATION OF SNOW DRIFTS, AND ITS PREVENTION OR CONTROL

MPF 1	5/30	PRACTICAL APPLICATIONS: GAS ~ LIQUID; ENGINEERING.

WITHOUT HEAT TRANSFER

- AIR-LIFT PUMPS
- USE OF GAS INJECTION FOR STIRRING MOLTEN GLASS
- GAS ~ LIQUID MOTION IN ELECTROLYTIC CELLS
- MOTION OF OIL AND GAS IN UNDER-OCEAN PIPE-LINES
- PAINT SPRAYING
- SEPARATION OF WATER DROPLETS FROM STEAM AT BOILER OUTLETS

WITH HEAT TRANSFER

- BOILING IN PIPES AND VESSELS
- CONDENSATION OF WATER IN STEAM TURBINES
- LOSS-OF-COOLANT ACCIDENTS AND RELATED STEAM ~ WATER PHENOMENA
- VAPORISATION AND BURNING OF LIQUID FUELS, IN FURNACES, GAS TURBINES, DIESEL ENGINES, ROCKET MOTORS
- CAVITATION PHENOMENA IN PUMPS AND TURBINES

MPF 1	6/30	PRACTICAL APPLICATIONS: GAS ~ LIQUID; THE ENVIRONMENT.

NATURAL

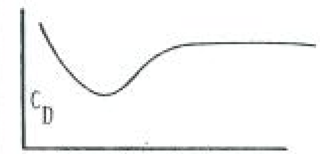
- FORMATION AND FALL OF RAIN
- WIND-BORNE OCEAN-SPRAY MOVEMENT
- UNDERGROUND FORMATION OF STEAM, LEADING TO GEYSER-ING PHENOMENA
- AIR ENTRAINMENT INTO WATERFALLS

MAN-MADE

- CARRY-OVER OF WATER DROP-LETS FROM COOLING TOWERS
- THE USE OF WATER JETS AND SPRAYS FOR EXTINGUISHING FIRES OF BUILDINGS AND FORESTS
- AIR CLEANSING BY AEROSOL INJECTION
- FLOW IN DECORATIVE FOUNTAINS
- SPRAY FORMATION BY HOVERCRAFT

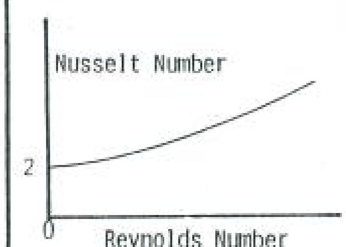
MPF 1	7/30	PHYSICAL-PROCESS REVIEW: INTER-PHASE FRICTION

- DRAG BETWEEN SMALL SOLID SPHERES AND LAMINAR SUPPORTING MEDIA IS REPRESENTED BY $C_D = C_D\ (Re)$.
- WHEN THE SPHERES ARE LIQUID, INNER RECIRCULATIONS SOMEWHAT DIMINISH DRAG. EFFECT DEPENDS ON VISCOSITY RATIO.
- WHEN SURFACE TENSION IS NOT VERY HIGH, DEPARTURES FROM SPHERICAL FORM OCCUR.



MPF 1	8/30	PHYSICAL-PROCESS REVIEW: HEAT TRANSFER

- FOR SMALL SPHERES OF HIGH CONDUCTIVITY, Nu = 2 AT VERY LOW Re.
- FOR MODERATE Re, $Nu \sim Re^{1/2}$.
- EFFECTS OF Pr, VISCOSITY RATIO, WEBER NUMBER (SURFACE TENSION) ARE ALSO PRESENT.
- SOLID PARTICLES (EG SAND GRAINS) MAY NOT BE SPHERICAL.



MPF 1	9/30	PHYSICAL-PROCESS REVIEW: MASS TRANSFER

- THE FLUID-SIDE (OUTER) RESISTANCE TO MASS TRANSFER FOLLOWS SIMILAR LAWS TO THOSE OF HEAT TRANSFER.
- IF ABSORPTION OF A GAS COMPONENT BY A LIQUID DROPLET IS IN QUESTION, THE DIFFUSIONAL RESISTANCE OF THE LIQUID IS IMPORTANT AND DEPENDS ON Sc, THE CIRCULATION, ETC.
- WHEN THE PARTICLE IS A POROUS CATALYST, OR A SUBSTANCE LIKE CHARCOAL, DIFFUSION INTO THE INTERIOR THROUGH THE PORES MAY HAVE TO BE CONSIDERED.
- FOR PULVERIZED-COAL BURNING, GUN-PROPELLANT COMBUSTION, ETC, DETAILED CHEMICAL-KINETIC PHENOMENA AFFECTING THE MASS-TRANSFER RATE.

MPF 1	10/30	PHYSICAL-PROCESS REVIEW: COALESCENCE

- DROPLETS COLLIDING WITH ONE ANOTHER MAY COMBINE TO FORM A SINGLE LARGER DROPLET.
- BUBBLES MAY ALSO COMBINE IN THIS WAY.
- COLLISION IS NOT ENOUGH: THERE MAY BE RESISTANCES TO COALESCENCE RESULTING FROM THE PRESENCE OF SURFACE-ACTIVE AGENTS.
- SOLID PARTICLES MAY ALSO COLLIDE AND STAY TOGETHER. THE PROCESS IS CALLED "FLOCCULATION".
- ELECTRICAL FORCES AFFECT COALESCENCE.

MPF 1	11/30	PHYSICAL-PROCESS REVIEW: OTHER MECHANICAL EFFECTS

- LARGE DROPLETS, MOVING AT LARGE VELOCITY RELATIVE TO A GAS, BREAK UP INTO SMALLER ONES. (SURFACE TENSION HAS BEEN UNABLE TO MAINTAIN A NEAR-SPHERICAL FORM).
- LARGE BUBBLES RISING THROUGH LIQUID BEHAVE SIMILARLY.
- TURBULENCE IN THE SUPPORTING FLUID EXAGGERATES THE PROCESSES, BECAUSE IT INCREASES RELATIVE VELOCITIES.
- THE DISRUPTION PROCESS IS ESPECIALLY IMPORTANT IN "ATOMISATION" OF LIQUID FUELS.
- PRESSURE INTERACTION: A PACKING-DENSITY LIMIT MAY BE REACHED.

MPF 1	12/30	PHYSICAL-PROCESS REVIEW: BOUNDARY EFFECTS

- WHEN SOLID SURFACES (EG HEAT-TRANSFER TUBES) ARE PRESENT IN THE DOMAIN, THE FRICTION, HEAT TRANSFER, ETC, OF EACH MOVING PHASE WITH THESE FIXED SURFACES REQUIRES CONSIDERATION.
- FIXED SURFACES DAMP OUT FLUID MOTION AND SO ALLOW "SETTLING", AND COHESION VIA SURFACE TENSION.
- THEREFORE, MUD LAYERS FORM ON THE BOTTOMS OF RIVERS; LIQUID FILMS ADHERE TO TUBE WALLS; DESERTS ARE NOT IN CONSTANT MOTION, ETC.

MPF 1	$\frac{13}{30}$	THE MATHEMATICAL PROBLEM: MEANS OF DESCRIPTION

- CHARACTERISATION OF MIXTURE COMPONENTS:
  - LET LOCAL TWO- (OR MULTI-) PHASE MIXTURE BE DIVIDED INTO GROUPS, CHARACTERISED BY: SIZE, SHAPE, COMPOSITION, TEMPERATURE, VELOCITY, ETC.
  - LET THE INDEX k DENOTE A PARTICULAR GROUP IN THIS "MULTI-DIMENSIONAL SPACE".
- LET r_k BE THE FRACTION OF ALL VOLUME OCCUPIED BY THE k'th GROUP. (NB $\sum_{all\ k} r_k = 1$, IF THE FIXED PHASE (POROUS SOLID) IS GIVEN A k VALUE).
- THE AVERAGE VALUE OF A PROPERTY ϕ OF THE MIXTURE IS GIVEN BY: $\bar{\phi} = \sum_k \rho_k \phi_k r_k / \sum_k \rho_k r_k$.

MPF 1	$\frac{14}{30}$	THE MATHEMATICAL PROBLEM: DIFFERENTIAL EQUATIONS, 1

THE VARIATION WITH SPACE AND TIME OF THE PROPERTY ϕ_k IS GIVEN BY:

$$\frac{\partial}{\partial t} (\rho_k r_k \phi_k) + \mathrm{div}\, (\rho_k r_k \vec{u}_k \phi_k - \Gamma_k r_k \,\mathrm{grad}\, \phi_k) = S_{k_\phi}$$

HERE $\vec{u}_k$ IS THE VELOCITY VECTOR OF THE GROUP;
Γ_k IS A DIFFUSION COEFFICIENT (IF APPROPRIATE; EXAMINATION IS REQUIRED IN EACH CASE);
S_{k_ϕ} IS THE SOURCE TERM (SEE NEXT SLIDE).
$\rho \equiv$ DENSITY;
$t \equiv$ TIME;
div $\equiv$ NET OUTFLOW PER UNIT VOLUME.

MPF 1	$\frac{15}{30}$	THE MATHEMATICAL PROBLEM: DIFFERENTIAL EQUATIONS, 2

- S_k ACCOUNTS FOR:
  - INTERNAL CHANGES WITHIN THE GROUP, EG BY CHEMICAL REACTION.
  - INTERACTIONS BETWEEN PHASES (THEN $\sum_k S_k = 0$, POSSIBLY).
  - EXTERNAL EFFECTS, EG THERMAL RADIATION, PRESSURE GRADIENT.
- NOTE: OFTEN IT IS USEFUL TO REGARD THE PRESSURE AS A VARIABLE SHARED BY ALL THE GROUPS. THEN, MOMENTUM SOURCES ARE OF FORM: $- r_k \vec{i}_j . \mathrm{grad}\, p$.

MPF 1	$\frac{16}{30}$	TWO-PHASE HYDRODYNAMICS: A SIMPLE SAND-AIR PROBLEM

- THE MATERIALS:
  - LET THERE BE A CONTINUOUS FLUID PHASE, EG AIR, AND A SUSPENDED SOLID-PARTICULATE PHASE, EG SAND.
  - LET THE SAND PARTICLES BE UNIFORM IN SIZE, AND SPHERICAL.
- THE DOMAIN: LET BOTH MATERIALS FLOW WITHIN A DOMAIN WITHOUT A POROUS FIXED PHASE.
- THE PROBLEM: LET THE TASK BE TO ESTABLISH THE VELOCITY COMPONENTS OF THE PHASES, u, v, w, U, V, W, THE PRESSURE, p, AND THE VOLUME FRACTIONS r, R, AT ALL POINTS AND TIMES.

MPF 1	17/30	TWO-PHASE HYDRODYNAMICS: THE DIFFERENTIAL EQUATIONS

- CONTINUITY: $\frac{\partial r}{\partial t} + \text{div}\,(r\vec{u}) = 0$

  $\frac{\partial R}{\partial t} + \text{div}\,(R\vec{U}) = 0$

  NOTE: THE DENSITY-FREE FORM IS USED, FOR SIMPLICITY. NO TIME-AVERAGING (WHICH WOULD INTRODUCE $\text{div}\,\overline{r'\vec{u}'}$, ETC) HAS BEEN CARRIED OUT.

- MOMENTUM: $\frac{\partial(\rho_\ell\, r\, u_j)}{\partial t} + \text{div}\,(\rho_\ell\, r\vec{u}\, u_j) = s_j$

  NOTES:
  - $\rho_\ell \equiv \rho$ OF LIGHT PHASE.
  - u_j IS ONE OF THREE VELOCITY COMPONENTS.
  - THERE ARE SIMILAR EQUATIONS FOR THE HEAVY PHASE.
  - s_j INCLUDES: $-r\,\vec{i}_j.\text{grad}\,p$, INTER-PHASE FRICTION, AND ORDINARY SHEAR-STRESS TERMS.

MPF 1	18/30	THE IPSA METHOD (INTER-PHASE SLIP ANALYSER) FINITE-DOMAIN EQUATIONS, 1

- FDE's FOR CONTINUITY: $r_P = \Sigma a_n\, r_n \,/\, \Sigma b_n$

  $R_P = \Sigma A_n\, R_n \,/\, \Sigma B_n$

- NOMENCLATURE:
  - a's AND b's ARE VELOCITY-AREA PRODUCTS, OR CELL VOLUMES DIVIDED BY TIME INTERVAL.
  - SUBSCRIPT P DENOTES A GRID POINT, SUBSCRIPT n ITS NEIGHBOUR.
  - a AND A REPRESENT INWARD-DIRECTED FLOWS.
  - b AND B REPRESENT OUTWARD-DIRECTED FLOWS.
- NOTES:
  - UPWIND DIFFERENCING IS EMPLOYED.
  - $r_P + R_P = 1$
  - FDE's FOR VELOCITY COMPONENTS CAN ALSO BE DERIVED.
  - THE CONVENTIONAL "STAGGERED GRID" IS APPROPRIATE.

MPF 1	19/30	IPSA: FINITE-DIFFERENCE EQUATIONS, 2

- FOR SINGLE-PHASE FLOW ($r = 1$, $R = 0$, SAY), THE CONTINUITY EQUATION REDUCES (IN PRESENT UNIFORM-ρ PROBLEM) TO:

  $\Sigma a_n = \Sigma b_n$, IE INFLOW = OUTFLOW.

- IN GENERAL, HOWEVER, $\Sigma a_n \neq \Sigma b_n$.

  THIS IS A DIFFERENCE FROM SINGLE-PHASE FLOW; IT ALLOWS r_P, FOR EXAMPLE, TO EXCEED EVERY ONE OF ITS NEIGHBOURS, ON OCCASION.

- HOWEVER, THE a's, b's, A's AND B's MUST OBEY THE JOINT CONTINUITY EQUATION:

  $\epsilon(r_P\Sigma b_n - \Sigma a_n r_n) + (1-\epsilon)\,(R_P\Sigma B_n - \Sigma A_n R_n) = 0,$

  WHERE ϵ IS ANY WEIGHTING FACTOR, CHOSEN AT WILL.

MPF 1	20/30	IPSA: SOLUTION PROCEDURE, 1

- GENERAL REMARKS:
  - THE PROCEDURE IS SIMILAR TO "SIMPLE", (PATANKAR-SPALDING, 1972), IN PROCEEDING BY WAY OF GUESSES OF, AND CORRECTIONS TO, THE PRESSURE (OR VELOCITY) FIELDS.
  - THE DIFFERENCE LIES IN THE CALCULATION OF THE PRESSURE (OR VELOCITY) CORRECTIONS FROM THE JOINT CONTINUITY EQUATION.
  - THE PROCEDURE REDUCES TO THE SINGLE-PHASE FORM WHEN r OR R BECOMES VERY SMALL.
  - NOT ALL REFINEMENTS ARE DESCRIBED IN THE FOLLOWING DESCRIPTION

MPF 1	$\frac{21}{30}$	IPSA: SOLUTION PROCEDURE, 2

1. SOLVE A PHASE-CONTINUITY EQUATION FOR r_P (SAY).
2. OBTAIN THE OTHER VOLUME FRACTION FROM: $r_P + R_P = 1$.
3. GUESS PRESSURE FIELD (p_*) AND SOLVE FOR ASSOCIATED VELOCITIES: $u_*, v_*, w_*, U_*, V_*, W_*$.
4. EVALUATE ERRORS IN THE JOINT CONTINUITY EQUATION, AND OBTAIN EQUATION FOR NECESSARY CHANGES TO THE COEFFICIENTS: a', b', A', B' WITH $\varepsilon = 0.5$ (A COMMON, BUT NOT OPTIMAL CHOICE)

$$r_P \sum b'_n + R_P \sum B'_n - \sum a'_n r_n - \sum A'_n R_n =$$

$$-r_P \sum b_n - R_P \sum B_n + \sum a_n r_n + \sum A_n R_n$$

MPF 1	$\frac{22}{30}$	IPSA: SOLUTION PROCEDURE, 3

5. OBTAIN PRESSURE CORRECTION p'_P FROM ABOVE EQUATION (DETAILS BELOW).
6. MAKE CORRESPONDING CHANGES TO VELOCITIES: u', v', w', U', V', W'.
7. RETURN TO 1 AND ITERATE UNTIL CORRECTIONS ARE SMALL.

DETAILS OF p' EQUATION:

- $a', b' \propto u', v'$ or w'; $A', B', \propto U', V'$ or W'.
- $u' = \frac{\partial u}{\partial p_{left}} p'_{left} + \frac{\partial u}{\partial p_{right}} p'_{right}$.

MPF 1	$\frac{23}{30}$	IPSA: PRESSURE-CORRECTION EQUATION, 1

- SIMILAR EQUATIONS EXIST FOR v', w', U', V', W'.
  THEY ARE DERIVED BY DIFFERENTIATION OF THE FDE's FOR MOMENTUM, WITH NEGLECT OF ALL BUT LEADING TERMS.
- THE CORRECTION FORM OF THE JOINT CONTINUITY EQUATION THUS BECOMES:

$$p'_P \sum \beta_n + \sum \alpha_n p'_n = -r_P \sum b_n - R_P \sum B_n + \sum a_n r_n + \sum A_n R_n$$

- SOLUTION OF THIS EQUATION SET, OVER THE WHOLE FIELD, LEADS TO PRESSURE CORRECTIONS, AND CONSEQUENTLY TO VELOCITY CORRECTIONS.
  THESE CAN THEN BE **ADDED** TO THE GUESSED VALUES.

MPF 1	$\frac{24}{30}$	IPSA: PRESSURE-CORRECTION EQUATION, 2

A SPECIAL PROBLEM WITH HIGH INTER-PHASE TRANSFER:

- MOMENTUM EQUATIONS CAN BE WRITTEN:

$$u_P = (\sum a_1 u_1 + s + f U_P) / (\sum a_1 + f)$$

$$U_P = (\sum A_1 U_1 + S + F u_P) / (\sum A_1 + F).$$

- FOR HIGH INTER-PHASE TRANSFER, f AND F ARE LARGE; THEN THE ADJUSTMENTS ARE SMALL IF u_P AND U_P ARE SOLVED SEPARATELY.
- SOLUTION (PEA ≡ PARTIAL ELIMINATION ALGORITHM):
- ELIMINATE U_P FROM FIRST EQUATION, u_P FROM SECOND, TO GIVE:

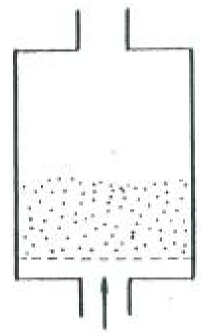
$$u_P = (\sum a^*_1 u_1 + s)/\sum a^*_1$$

$$U_P = \text{SIMILARLY.}$$

- USE THESE EQUATIONS FOR CONNECTING u', U' WITH p'.

MPF 1 | 25/30 | IPSA APPLICATIONS: 1D UNSTEADY

- FLUIDIZATION:
  - A BED OF SAND SOAKED IN WATER IS AT REST. THEN WATER STARTS TO FLOW IN FROM THE BOTTOM.
  - AT LOW FLOW, THE SAND REMAINS IN POSITION. WHEN THE FLOW, AND PRESSURE GRADIENT, INCREASE, THE SAND STARTS TO MOVE. A STEADY FLUIDIZED STATE CAN BE ARRIVED AT.
  - WHEN THE VELOCITY EXCEEDS THE "SETTLING VELOCITY" OF THE SAND, IT ALL FLOWS OUT THROUGH THE TOP.
- SEDIMENTATION: THIS IS WHAT HAPPENS WHEN THE FLOW IS STOPPED: THE SAND SETTLES.



MPF 1 | 26/30 | IPSA APPLICATIONS CONTINUED: 1D UNSTEADY COMPRESSIBLE

- THE 1D SAND-FILLED SHOCK TUBE. WHEN THE DIAPHRAGM BREAKS, THE GAS EXPANDS. IT CARRIES THE SAND WITH IT TO SOME EXTENT.

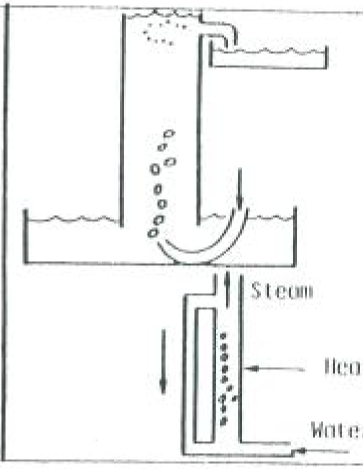
- NOTES:
  - THIS IS AN EASY EXPERIMENT TO PERFORM; AND IT WOULD PROVIDE AN EXCELLENT TEST OF THE CALCULATION PROCEDURE.
  - IT IS RELEVANT TO IMPORTANT GAS ~ LIQUID FLOW PHENOMENA, EG THE PIPE-BURST PHENOMENON.
  - VARIATIONS OF THE BOUNDARY CONDITIONS CAN PRODUCE OTHER INTERESTING PHENOMENA, EG THOSE WITH STEADY-STATE SOLUTIONS.

MPF 1 | 27/30 | IPSA APPLICATIONS CONTINUED: 1D STEADY

- THE AIR-LIFT PUMP.

  IN THIS PROBLEM, THE INTER-PHASE-FRICTION COEFFICIENTS MUST BE TREATED AS UNKNOWNS, TO BE DEDUCED FROM EXPERIMENTAL DATA.
- BOILER-WATER CIRCULATION.

  THIS PROBLEM IS SIMILAR; BUT THERE ARE SOURCES OF VAPOUR AND SINKS OF LIQUID TO BE ACCOUNTED FOR.



MPF 1 | 28/30 | IPSA APPLICATIONS CONTINUED: 2D

- EACH OF THE PROBLEMS OF SLIDES 25, 26, AND 27 CAN BE MADE 2D BY INTRODUCTION OF BAFFLES, FLOW ENLARGEMENTS, ETC; EG THE BAFFLED SHOCK TUBE.

- SEVERAL 2D PARABOLIC PROBLEMS ARE WORTHY OF INITIAL STUDY, EG:
  - THE PARTICLE-LADEN JET.
  - THE SHOWER-BATH FLOW.
  - THE RISING PLUME OF BUBBLES IN A LIQUID.
- GENMIX 2P WILL HANDLE THESE.

MPF 1 | 29/30 | IPSA APPLICATIONS: MORE COMPLEX EXAMPLES

- TRANSPORT OF SEDIMENT IN A HELICAL PIPE (3D PARABOLIC; RELEVANT TO FLOW IN RIVER BENDS).
- FALL-OUT FROM A COOLING-TOWER PLUME (3D PARABOLIC).
- THE FLUIDIZED-BED COAL COMBUSTOR (BUBBLE FORMATION IS ESSENTIAL) (2D OR 3D UNSTEADY).
- THE 2-PHASE FLOW OF STEAM AND WATER IN A NUCLEAR BOILER (3D STEADY ELLIPTIC).

MPF 1 | 30/30 | CONCLUDING REMARKS

- A CALCULATION PROCEDURE (IPSA) HAS BEEN DESCRIBED WHICH MAKES MULTI-PHASE PROBLEMS AS EASY TO SOLVE AS 1-PHASE ONES (ALBEIT MORE EXPENSIVE).
- DEMONSTRATIONS AND TESTS ARE NOW IN PROGRESS. 1D SITUATIONS ARE ESPECIALLY CONVENIENT, BUT 2D AND 3D ARE ALSO STUDIED.
- "IPSA" IS BEING ACTIVELY APPLIED TO THE COMPUTATION OF PIPE-LINE, GUN-BARREL AND STEAM-GENERATOR PROBLEMS.
- MAJOR UNCERTAINTIES AS TO THE INTER-PHASE TRANSFER COEFFICIENTS REQUIRE TO BE REMOVED BY EXPERIMENTAL RESEARCH.
- HOWEVER, THE PRACTICAL NEED IS TOO GREAT FOR THE OUTCOME OF THIS RESEARCH TO BE AWAITED.

MPF 2 | 1/15 | LECTURE 2: ONE-DIMENSIONAL UNSTEADY TWO-PHASE DISPERSED FLOW, WITHOUT PHASE CHANGE

CONTENTS:

- SCOPE OF LECTURE
- DIFFERENTIAL EQUATIONS
- AUXILIARY RELATIONS
- FINITE- DOMAIN EQUATIONS
- SOLUTION PROCEDURE
- RESULTS
- DISCUSSION
- REFERENCES

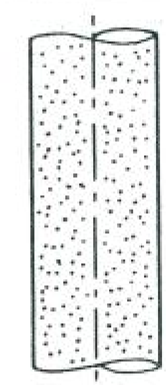
MPF 2 | 2/15 | EXEMPLIFICATION OF SCOPE OF LECTURE 2

PHENOMENA IN QUESTION:

- STEADY OR UNSTEADY FLOW;
- INTERPHASE AND WALL FRICTION;
- BODY FORCES IN FLOW DIRECTION;
- PRESSURE-DEPENDENCE OF DENSITIES.

RESTRICTIONS:

- NO VARIATIONS NORMAL TO FLOW DIRECTION;
- NO FLOW-DIRECTION DIFFUSION;
- DISCONTINUOUS PHASE CONSISTS OF IDENTICAL PARTICLES.



MPF 2	$\frac{3}{15}$	PARTIAL DIFFERENTIAL EQUATIONS FOR THE LIGHTER PHASE

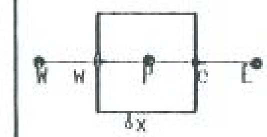
- CONTINUITY: $\frac{\partial r}{\partial t} + \frac{\partial}{\partial x}(ru) = \frac{r}{v}\frac{Dv}{Dt}$.
- MOMENTUM: $\frac{\partial}{\partial t}\left(\frac{ru}{v}\right) + \frac{\partial}{\partial x}\left(\frac{ru}{v}.u\right) = -uf_s + (U-u)f_I - r\frac{\partial p}{\partial x} - rgv^{-1}$.
- NOMENCLATURE:
  - r, u, v ≡ VOLUME FRACTION, VELOCITY AND SPECIFIC VOLUME OF LIGHT PHASE;
  - f_s, f_I ≡ WALL AND INTERPHASE FRICTION COEFFICIENTS;
  - g ≡ GRAVITATIONAL ACCELERATION;
  - p ≡ PRESSURE;
  - U ≡ VELOCITY OF HEAVY PHASE.

MPF 2	$\frac{4}{15}$	AUXILIARY RELATIONS

- VOLUME COMPATIBILITY: $r + R = 1$.
- FRICTION FACTORS: $f_s = v^{-1}|u|F_s(Re, \text{roughness})$.
  $f_I = v^{-1}|u - U|F_I(Re, \text{shape})$.
- COMPRESSIBILITY: v^{-1} = FUNCTION OF PRESSURE.
- DIFFUSION:
  IN PRINCIPLE, "LONGITUDINAL DIFFUSION" TERMS CAN ENTER THE EQUATIONS, EG $\frac{\partial}{\partial x}\left(\mathcal{D}\frac{\partial r}{\partial x}\right)$, TO ACCOUNT FOR THE FACT THAT, IN REALITY, u AND U ARE UNIFORM NEITHER IN CROSS-STREAM DISTANCE ("PROFILE" EFFECTS) NOR IN TIME (TURBULENCE EFFECTS). THIS IS IGNORED HERE, FOR SIMPLICITY.

MPF 2	$\frac{5}{15}$	FINITE-DOMAIN EQUATIONS, 1: CONTINUITY FOR THE LIGHT PHASE

- GRID: THIS IS "STAGGERED", THUS:



- EQUATION: THIS EMPLOYS "UPWIND INTERPOLATION" AND IS FULLY IMPLICIT, THUS:

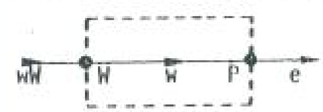
$$\big(r_P\,([-u_w] + [u_e]) - r_W\,[u_w] - r_E\,[-u_e]\big)\,\delta t + (r_P - r_{P-})\,\delta x - r_P\,(V_P/V_{P-} - 1)\,\delta x = 0.$$

(volume outflow; volume inflow; storage; compressibility)

- COMPACT FORM: $r_P = \dfrac{r_W[U_w] + r_E[-U_e] + r_{P-}\,\delta x/\delta t}{[-U_w] + [U_e] + (2 - V_P/V_{P-})\,\delta x/\delta t}$
- NOTE: (ε) ≡ LARGER OF ε AND ZERO

MPF 2	$\frac{6}{15}$	FINITE- DOMAIN EQUATIONS, 2: MOMENTUM FOR THE LIGHT PHASE

- GRID: NOTE CHANGE OF CONTROL VOLUME:



- EQUATION:

$$u_w = \frac{\left(u_{ww}\left[\frac{r}{v}u_w\right] + {}_e\left[-\frac{r}{v}u_p\right]\right) + r_w(p_W - p_P) + \left(\frac{r}{v\delta t}\,u_{w-} + f_I u_w - \frac{r}{I}g\right)\delta x}{\left(\left[-\frac{r}{v}u_w\right] + \left[\frac{r}{v}u_p\right]\right) + \left(\frac{r}{v\delta t} + f_s + f_I\right)\delta x}$$

- NOTES:
  - NODAL r/v's SHOULD BE USED AS VELOCITY MULTIPLIERS.
  - r/v FOR THE TRANSIENT TERM IS: $((r/v)_W + (r/v)_P)/2$.
  - ALTERNATIVE FORMULATIONS ARE PERMISSIBLE; BUT THEY SHOULD ALL ENSURE CELL-TO-CELL CONSISTENCY AND OVERALL MOMENTUM BALANCE.

MPF 2	$\frac{7}{15}$	FINITE- DOMAIN EQUATIONS, 3: APPRECIATION OF THE PROBLEM OF SOLUTION.

- NUMBER OF EQUATION SETS: 5 (FOR r, R, u, U, r + R).
- NUMBER OF UNKNOWNS: 5 (viz r, R, u, U, p).
- INITIAL CONDITIONS: VALUES OF r, R, u, U, p AT t = 0.
- BOUNDARY CONDITIONS: VARIOUS MIXES ARE POSSIBLE. TYPICAL ARE: r, R, u, U AT INFLOW BOUNDARIES; p AT OUTFLOW.
- SOME REQUIREMENTS OF THE SOLUTION PROCEDURE:
  - VALIDITY OVER WHOLE RANGE: $0 \leq r \leq 1$.
  - UNIFORMITY OF CONVERGENCE.
  - ECONOMY.
  - EXTENSIBILITY TO ARBITRARY NUMBER OF PHASES.

MPF 2	$\frac{8}{15}$	THE IPSA SOLUTION PROCEDURE, 1: OUTLINE (AN UNSYMMETRICAL VERSION).

- SOLUTION FOR R WITH IN-STORE VELOCITIES SATISFIES HEAVY-PHASE CONTINUITY.
- THEN r COMES FROM 1.0 - R.
- GUESSED (EG IN-STORE) PRESSURES LEAD TO APPROXIMATE VELOCITIES (u_*, U_*).
- SUBSTITUTION OF r, R, u_*, U_* INTO JOINT CONTINUITY REVEALS ERRORS IN THAT EQUATION.
- THESE ERRORS ARE RIGHT-HAND SIDES OF PRESSURE-CORRECTION EQUATIONS. SOLUTION YIELDS p'.
- p' VALUES LEAD TO ASSOCIATED u' AND U', WHICH ARE THEN ADDED TO u_* AND U_*.
- p' MAY BE ADDED TO p_* (SIMPLE) OR DISCARDED (SNIP).

MPF 2	$\frac{9}{15}$	THE IPSA SOLUTION PROCEDURE, 2: DETAILS OF HOW THE EQUATIONS ARE SOLVED

- ALTHOUGH LINE-BY-LINE PROCEDURES ARE FAST FOR LINEAR UNCOUPLED EQUATIONS, POINT-BY-POINT PROCEDURES ARE MORE FREE FROM DIVERGENCE TENDENCIES FOR COUPLED EQUATIONS.
- u AND U MUST BE ELIMINATED FROM EACH OTHER'S EQUATIONS WHEN INTER-PHASE FRICTION IS SIGNIFICANT; OTHERWISE CONVERGENCE IS SLOW.
- THE PARTIAL-ELIMINATION ALGORITHM (PEA, PANEL 1.24) IS USED FOR THIS PURPOSE.
- MORE ELABORATE ALGORITHMS CAN BE CONCEIVED, INVOLVING GREATER IMPLICITNESS (E.G. TWO-VARIABLE ALTERNATING-DIRECTION IMPLICIT), AND SOME HAVE BEEN TRIED; THEY ARE NOT RECOMMENDED.

MPF 2	$\frac{10}{15}$	THE IPSA PROCEDURE, 3: CONNEXIONS OF u', U' WITH p'.

- FROM THE u-EQUATION: $$u'_W = \frac{(P'_W - P'_P)\, r_W}{([-\frac{r}{v}u_W] + [\frac{r}{v}u_P]) + (\frac{r}{v\delta t} + f_I + f_S)\delta x}$$
- SIMILAR EXPRESSIONS EXIST FOR u'_e, U'_w, U'_e.
- THE CORRECTION FORM OF THE JOINT VOLUMETRIC-CONTINUITY (ξ=0.5) EQUATION IS: $$r_P([-u_W]' + [u_e]') + R_P([-U_W]' + [U_e]') - r_W[u_W]' - v_E[-u_e]' - R_W[U_W]' - R_E[-U_e]' =$$ = error associated with r's, R's, u_*'s and U_*'s.
- THE PRIMED VELOCITIES CAN NOW BE ELIMINATED.
- Note: Large f's imply small u's. PEA would cure this; but it is omitted here, for simplicity.

MPF 2 | 11/15 | THE IPSA SOLUTION PROCEDURE, 4: OBTAINING THE NEW VARIABLE FIELD

- IN "FAST-TRANSIENT" PROBLEMS, VARIATION OF v WITH p ALSO NEEDS TO BE ACCOUNTED FOR; THEN $-(r/v^2)(\partial v/\partial p)\ p'_p$ APPEARS ON THE LHS.
- SOLUTION BY TDMA FOR p' DISTRIBUTION NOW FOLLOWS.
- THEREAFTER, u', U', v' ARE COMPUTED, AND ADDED.
- IN "SIMPLE", p IS NOW CHANGED TO p_* + SOME FRACTION OF p'.
- IN "SNIP", p IS COMPUTED FROM ONE OR OTHER OF THE MOMENTUM EQUATIONS, OR FROM THEIR SUM.

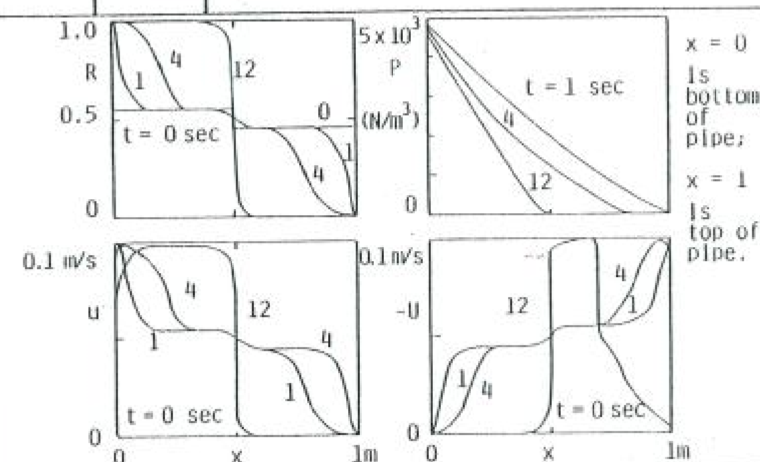
MPF 2 | 12/15 | A PARTICULAR EXAMPLE, 1: PROBLEM STATEMENT

- NATURE: SEDIMENTATION.
- FLUID PROPERTIES: DENSITY RATIO = 10 : 1
- GEOMETRY: VERTICAL PIPE OF 1.0M HEIGHT.
- INITIAL PROFILE OF : APPROXIMATELY 0.5 THROUGHOUT.
- INITIAL VELOCITIES: ZERO.
- WALL FRICTION FACTOR: ZERO.
- GRAVITY: NORMAL.
- INTERPHASE FRICTION: HIGH.
- TO FIND: r, R, u, U, p AS FUNCTIONS OF TIME.

MPF 2 | 13/15 | A PARTICULAR EXAMPLE, 2: DETAILS OF COMPUTATION

- COMPUTER CODE: "PLANT" (≡ PIPE-LINE ANALYSIS, TWO-PHASE). THIS CODE EMBODIES THE IPSA ALGORITHM IN THE FORM DESCRIBED ABOVE. IT IS WRITTEN IN FORTRAN.
- GRID: 42 PRESSURE NODES, 41 VELOCITY NODES, UNIFORMLY SPACED.
- TIME STEP: 0.5 SECONDS.
- NO OF ITERATIONS PER TIME STEP: 10.
- COMPUTER TIME: 0.05 SECONDS/GRID NODE/TIME STEP ON CDC 6500.
- REFERENCE: Y KUROSAKI AND D B SPALDING, 1979.

MPF 2 | 14/15 | A PARTICULAR EXAMPLE, 3: RESULTS



MPF 2	15/15	CONCLUDING REMARKS

- THE "IPSA" PROCEDURE, VALID FOR MULTI-PHASE FLOW, HAS BEEN SHOWN TO WORK FOR 2-PHASE FLOW.
- RESULTS ARE QUALITATIVELY PLAUSIBLE.
- NUMEROUS COMPUTATIONS ARE BEING MADE TO TEST:
  - PHYSICAL PLAUSIBILITY;
  - SENSITIVITY TO GRID SIZE, TIME STEP, ETC;
  - SENSITIVITY TO PHYSICAL INFLUENCES, eg f_i, f_s.
- IMMEDIATE USE FOR DESIGN PURPOSES IS HAMPERED BY LACK OF KNOWLEDGE OF THE f_I AND f_s FUNCTIONS.
- DIFFUSION TERMS CAN BE INCLUDED.

MPF 3	1/15	LECTURE 3: 2D STEADY DISPERSED FLOW

CONTENTS

- PRACTICAL EXAMPLES: (a) PARABOLIC
  (b) ELLIPTIC
- A PARABOLIC PROBLEM:
  - DESCRIPTION
  - COORDINATE SYSTEM
  - DIFFERENTIAL EQUATIONS
  - FINITE-DOMAIN EQUATIONS
  - SOLUTION PROCEDURE
- DISCUSSION AND CONCLUDING REMARKS

MPF 3	2/15	PRACTICAL EXAMPLES, 1: PARABOLIC FLOWS

- EXHAUST GASES FROM A ROCKET COMBUSTION CHAMBER CARRY SUSPENDED PARTICLES THROUGH THE NOZZLE; THESE ACCELERATE LESS RAPIDLY THAN THE GAS, AND IMPAIR PERFORMANCE.
- SNOW FALLS THROUGH AIR IN RAPID HORIZONTAL STEADY MOTION (WIND); THE TRAJECTORY OF THE SNOW FLAKES IS TO BE CALCULATED.
- AIR CONTAINING SUSPENDED WATER DROPLETS FLOWS THROUGH THE SPACE BETWEEN TWO CORRUGATED PLATES; THE FLOW DIRECTION IS NORMAL TO THE RIDGES. THE IMPINGEMENT OF DROPLETS ON THE PLATES IS TO BE CALCULATED.

MPF 3 | 3/15 | PRACTICAL EXAMPLES, 2: FURTHER PARABOLIC EXAMPLES

- A STREAM OF AIR CARRYING SAND PARTICLES FLOWS FROM THE NOZZLE OF A SAND-BLASTING DEVICE INTO THE ATMOSPHERE. THE SAND DISTRIBUTION IN THE RESULTING JET IS TO BE COMPUTED.
- A COAL-AIR STREAM MIXES WITH HOT FURNACE GASES IN THE FORM OF A JET; THE HEATING LENGTH OF THE PARTICLES IS TO BE PREDICTED.
- A WIDE RIVER INCREASES IN DEPTH, CONSEQUENTLY REDUCING ITS VELOCITY; SOLID PARTICLES SUSPENDED IN THE FAST-FLOWING REACHES NOW SINK TO THE BOTTOM.

MPF 3 | 4/15 | PRACTICAL EXAMPLES, 3: SOME ELLIPTIC EXAMPLES

- WIND-BORNE SNOW PILES UP BEHIND A FENCE AS A CONSEQUENCE OF THE RECIRCULATING FLOW WHICH IT FORMS.
- LIQUID FUEL,SPRAYED INTO AN ILL-DESIGNED GAS-TURBINE COMBUSTION CHAMBER,HITS THE FLAME-TUBE WALLS.
- A CYCLONE SEPARATOR SPLITS A DUSTY-GAS STREAM INTO A CLEAR-GAS STREAM AND A PILE OF DUST.
- MOLTEN GLASS IS CAUSED TO CIRCULATE IN A GLASS TANK BY THE INJECTION OF GAS BUBBLES FROM A ROW OF NOZZLES NEAR THE FLOOR.

MPF 3 | 5/15 | A PARABOLIC PROBLEM: QUALITATIVE DESCRIPTION

- A WIDE STREAM OF AIR FLOWS STEADILY OVER A GENTLY UNDULATING SURFACE, CARRYING PARTICLES OF UNIFORM-SIZED DUST.
- A LONGITUDINAL PRESSURE DISTRIBUTION (EG CAUSED BY CONFINEMENT OF THE STREAM BY AN UPPER SURFACE) CAUSES THE LONGITUDINAL AIR VELOCITY TO VARY WITH DISTANCE.
- GRAVITY CAUSES THE DUST PARTICLES TO FALL DURING FLIGHT; BUT PARTICLE-AIR FRICTION LIMITS THE PROCESS.
- THE LONGITUDINAL VELOCITY OF THE DUST CHANGES WITH THAT OF THE AIR, BUT NOT SO RAPIDLY.
- THE DUST AND THE AIR MAY BE AT DIFFERENT TEMPERATURES.
- TURBULENT MIXING PROCESSES WILL BE NEGLECTED.

MPF 3 | 6/15 | A PARABOLIC PROBLEM: MATHEMATICAL ASSESSMENT

- WERE IT NOT FOR THE VELOCITY DIFFERENCES, THE FLOW WOULD BE TREATED AS A BOUNDARY LAYER, PREDICTABLE BY (EG) "GENMIX".
- VERTICAL PRESSURE VARIATIONS WOULD BE REGARDED AS NOT LARGE ENOUGH TO AFFECT LONGITUDINAL VELOCITIES; AND THEY WOULD NOT BE COMPUTED.
- WITH TWO PHASES PRESENT, THE EFFECT ON LONGITUDINAL VELOCITIES OF VERTICAL PRESSURE VARIATIONS MAY STILL BE IGNORED; BUT, THE VARIATIONS MUST BE COMPUTED, BECAUSE THEY AFFECT THE SEPARATION BETWEEN THE PHASES.
- VERTICAL-MOMENTUM EQUATIONS MUST THEREFORE BE SOLVED AS WELL AS HORIZONTAL ONES, FOR BOTH PHASES, DESPITE THE OTHERWISE PARABOLIC CHARACTER OF THE FLOW.
- MARCHING INTEGRATION WILL HOWEVER STILL BE PERMISSIBLE.

MPF 3	7/15	THE PARABOLIC PROBLEM: MATHEMATICAL DESCRIPTION (PLANE FLOW)

- COORDINATE SYSTEM: x, ψ_1, where $\psi_1 \equiv \psi_{1o} + \int_0^y \rho_1 u_1 r_1 dy$
- DIFFERENTIAL EQUATIONS FOR THE FIRST PHASE:
  - x-momentum: $\frac{\partial u_1}{\partial x} = \frac{\partial}{\partial \psi_1}\left(\mu_1 \frac{\partial u_1}{\partial y}\right) - \frac{1}{\rho_1 u_1}\frac{\partial \bar{p}}{\partial x} + \frac{(u_2 - u_1)F}{\rho_1 u_1 r_1}$
  - y-momentum: $\frac{\partial v_1}{\partial x} = - \frac{1}{\rho_1 u_1}\frac{\partial p}{\partial y} + \frac{(v_2 - v_1)F}{\rho_1 u_1 r_1}$
  - enthalpy: $\frac{\partial h_1}{\partial x} = \frac{\partial}{\partial \psi_1}\left(\frac{\mu_1}{\sigma_1}\frac{\partial h_1}{\partial y}\right) + \left(\frac{h_2}{c_2} - \frac{h_1}{c_1}\right)\frac{H}{\rho_1 u_1 r_1}$
- NOMENCLATURE FOR FIRST-PHASE QUANTITIES:
  $\psi_1 \equiv$ stream function; $u_1, v_1 \equiv$ x,y velocities; $\rho_1 \equiv$ density
  $\mu_1 \equiv$ viscosity ; $h_1 \equiv$ enthalpy ;
  $c_1 \equiv$ specific heat ; $\sigma_1 \equiv$ Prandtl Number.

MPF 3	8/15	THE PARABOLIC PROBLEM: MATHEMATICAL DESCRIPTION CONTINUED

- DIFFERENTIAL EQUATIONS FOR THE SECOND PHASE:
  - x-momentum: $\frac{\partial u_2}{\partial x} - \left(\frac{v_1}{u_1} - \frac{v_2}{u_2}\right)\frac{\partial u_2}{\partial y} = -\frac{1}{\rho_2 u_2}\frac{\partial \bar{p}}{\partial x} + \frac{(u_1 - u_2)F}{\rho_2 u_2 r_2}$
  - y-momentum: $\frac{\partial v_2}{\partial x} - \left(\frac{v_1}{u_1} - \frac{v_2}{u_2}\right)\frac{\partial v_2}{\partial y} = -\frac{1}{\rho_2 u_2}\frac{\partial p}{\partial y} + \frac{(v_1 - v_2)F + (\rho_1 - \rho_2)g}{\rho_2 u_2 r_2}$
  - enthalpy: $\frac{\partial h_2}{\partial x} - \left(\frac{v_1}{u_1} - \frac{v_2}{u_2}\right)\frac{\partial h_2}{\partial y} = \left(\frac{h_1}{c_1} - \frac{h_2}{c_2}\right)\frac{H}{\rho_2 u_2 r_2}$
  - continuity: $\frac{\partial}{\partial x}(\rho_2 u_2 r_2) - \frac{\partial}{\partial y}\left[\left(\frac{v_1}{u_1} - \frac{v_2}{u_2}\right)(\rho_2 u_2 r_2)\right] = 0$
- NOTES: • DIFFERENCE FROM 1ST-PHASE EQUATIONS RESULTS FROM USING 1ST-PHASE STREAM FUNCTION AS INDEPENDENT VARIABLE.
- F & H REPRESENT INTERPHASE FRICTION & HEAT-TRANSFER FUNCTIONS.
- VISCOSITY AND HEAT CONDUCTION HAVE BEEN NEGLECTED.

MPF 3	9/15	THE PARABOLIC PROBLEM: DISCUSSION

- EVEN WHEN F AND H ARE ZERO, THE 1ST- AND 2ND-PHASE EQUATIONS ARE LINKED THROUGH r_1 AND r_2, WHICH SUM TO UNITY.
- A CHANGE IN r_2, AND SO IN r_1 ALSO, INFLUENCES THE INCLINATION OF THE CONSTANT-ψ_1 LINES, THROUGH:
  $y = \int_0^{\psi_1} (\rho_1 u_1 r_1)^{-1} \, d\psi_1$.
- THE INCLINATION CHANGE REQUIRES A CHANGE IN v_1.
- THE $\partial p/\partial y$'s MUST ALTER TO BRING THIS ABOUT.
- THE CHANGING $\partial p/\partial y$'s ALTER v_2 ALSO.
- AS A CONSEQUENCE, THE r_2's CHANGE FURTHER.
- THE SOLUTION PROCEDURE MUST TAKE THESE INTERACTIONS INTO ACCOUNT.

MPF 3	10/15	THE PARABOLIC PROBLEM: MARCHING-INTEGRATION PROCEDURE

- PRELIMINARY NOTE: SOLUTION PROCEDURES ARE NOT UNIQUE, ALTHOUGH SOLUTIONS MAY BE. THE FOLLOWING IS IN "GENMIX 2P".
- SET UP AN $x \sim \psi_1$ GRID, AND EXPRESS ALL DIFFERENTIAL EQUATIONS IN FINITE-DIFFERENCE FORM (NOTE THAT v NODES LIE ON THE BOUNDARIES BETWEEN "REGULAR" CELLS).
- GUESS DOWNSTREAM r_1 DISTRIBUTION (EG SAME AS UPSTREAM).
- SOLVE FOR u_1, h_1 DOWNSTREAM; OBTAIN y's AND THENCE v's.
- OBTAIN $p(y)$ FROM VERTICAL MOMENTUM EQUATION.
- SOLVE FOR DOWNSTREAM v_2, u_2, h_2, r_2. HENCE GET r_1.
- COMPARE NEW r_1 WITH GUESSED VALUE.
- MAKE CORRECTIONS AND REPEAT.

MPF 3	11/15	ALTERNATIVE r_1-CORRECTION PROCEDURES: METHOD 1: DIRECT SUBSTITUTION

- METHOD: SIMPLY USE THE NEW r_1's AS THE GUESSED VALUES FOR THE START OF THE NEW SOLUTION LOOP.
- ADVANTAGES: SIMPLICITY, AND CONSEQUENT ECONOMY.
- DISADVANTAGES: POSSIBLE NON-CONVERGENCE, WHEN THE r_1 CHANGES ARE LARGE.
- DISCUSSION: THIS METHOD CAN BE EXPECTED TO WORK SATISFACTORILY WHEN THE r_1 VALUES ARE CLOSE TO 1 (IE WHEN THE DUST CONCENTRATION IS SMALL).
- REMEDY FOR DIVERGENCE: USE UNDER-RELAXATION (UR). BUT WHAT VALUE SHOULD THE UR FACTOR HAVE?

MPF 3	12/15	ALTERNATIVE r_1-CORRECTION PROCEDURES: METHOD 2: COMPUTED UNDER-RELAXATION

- METHOD: START NEXT ITERATION LOOP WITH r_1's GIVEN BY

  $r_{1,old} + \alpha(r_{1,new} - r_{1,old})$

  WHERE α IS AN UNDER-RELAXATION FACTOR, CONVENIENTLY TAKEN AS: $\alpha \approx r_{1,old}$
- CONSEQUENCES:
  - UNDER-RELAXATION IS SLIGHT FOR DILUTE SUSPENSIONS $r_1 \not< 1$.
  - HEAVY UNDER-RELAXATION OCCURS AS r_2 APPROACHES UNITY.
- ORIGIN:
  - PRESSURE-CORRECTION PROCEDURE REDUCES TO THIS FOR SMALL δx, AND FOR v_1 AND v_2 TIGHTLY CONNECTED AS BY BOUNDARY CONDITIONS OR FRICTION.

MPF 3	13/15	ALTERNATIVE r_1-CORRECTION PROCEDURES: METHOD 3: USE OF PRESSURE CORRECTIONS

- METHOD: DEVISE AND SOLVE PRESSURE-CORRECTION EQUATION, "DRIVEN" BY ERRORS: $1 - r_1 - r_2$.
- ORIGIN:
  - FROM GEOMETRY AND PHASE-1 CONTINUITY:
    $Y'_1/Y = r_{21}'/r_{11} = (\delta x/Y_1)((v'_1/u_1)_{i+\frac{1}{2}} - (v'_1/u_1)_{i-\frac{1}{2}})$

  WHERE: $Y_1 \equiv$ DOWNSTREAM CELL AREA, PRIME $\equiv$ CORRECTION.
  - FROM PHASE-1 CONTINUITY, r_2' = LINEAR FUNCTION OF v_1', v_2', Y'.
  - FROM MOMENTUM, v_1' AND v_2' = LINEAR FUNCTIONS OF PRESSURE INCREMENTS p'.
- RESULTS:
  - AN EQUATION, SOLUBLE BY TDMA: $Dp_i' = Ap'_{i+1} + Bp'_{i-1} + C$
  - r_2' IS THEN OBTAINABLE FOR ALL I.

MPF 3	14/15	DISCUSSION OF THE SOLUTION PROCEDURE FOR THE PARABOLIC PROBLEM

- THE PROCEDURE EMBODIES THE BASIC "IPSA" IDEA OF BASING CORRECTIONS ON ERRORS IN THE JOINT CONTINUITY EQUATION.
- THE PROCEDURE DIFFERS IN DETAIL FROM THAT OF LECTURE 2 MAINLY BECAUSE OF THE DIFFERENT COORDINATE SYSTEM.
- IN PARTICULAR, THE METHOD IS UNSYMMETRICAL:

  PHASE 1 IS TREATED DIFFERENTLY FROM PHASE 2, IN THAT ITS STREAM FUNCTION IS CHOSEN AS A COORDINATE.
- IT IS ALSO POSSIBLE (AND MAY SOMETIMES BE CONVENIENT) TO USE THE MIXTURE STREAM FUNCTION INSTEAD, OR AS WELL.

MPF 3	15/15	EXTENSION TO 2D ELLIPTIC (EG RECIRCULATING) FLOWS

- METHOD 1 IS PARTICULARLY EASY TO APPLY AS A "TWO-PHASE UPDATE" FOR AN EXISTING SINGLE-PHASE COMPUTER CODE. IT WORKS WELL IF THE FIRST PHASE (FOR WHICH THE r DISTRIBUTION IS GUESSED) IS THE LIGHTER ONE, AND IN LARGE EXCESS.
- METHOD 2 (COMPUTED UNDER-RELAXATION) EXTENDS THE RANGE OF SUCH AN UPDATED CODE INTO REGIONS OF SMALLER r_1 AND LARGER ρ_1.
- IN GENERAL THE EQUIVALENT OF METHOD 3 IS REQUIRED. IT HAS ALREADY BEEN INTRODUCED IN LECTURES 1 AND 2, AND WILL BE FURTHER DISCUSSED BELOW.

MPF 4	1/15	LECTURE 4: THE 'GENMIX 2P' COMPUTER CODE: INTRODUCTION

CONTENTS

- SCOPE OF 'GENMIX 2P'
- STRUCTURE OF 'GENMIX 2P'
- MAJOR VARIABLES
- DATA INPUT
- OUTPUT OF RESULTS
- A PARTICULAR EXAMPLE

NOTE: THE PURPOSE OF THE LECTURE IS TO PERMIT EARLY USE OF 'GENMIX 2P'. FULLER EXPLANATION FOLLOWS IN LECTURE 8.

MPF 4	2/15	SCOPE OF 'GENMIX 2P': STEADY-STATE MODE (TRNSNT = . FALSE.)

- GENMIX 2P SOLVES STEADY TWO-DIMENSIONAL PARABOLIC DISPERSED-FLOW PROBLEMS, WITH INTERPHASE SLIP.
- VISCOUS AND HEAT-CONDUCTION EFFECTS ARE ALLOWED IN THE FIRST (CONTINUOUS) PHASE, BUT NOT IN THE SECOND ONE.
- INTERPHASE MOMENTUM AND HEAT TRANSFER ARE ALLOWED FOR, BUT NOT MASS TRANSFER.
- GENMIX 2P HAS BEEN DERIVED BY ADDING TO GENMIX 0, PUBLISHED AS APPENDIX F OF REFERENCE 1.
- GENMIX 0 ITSELF WAS DERIVED BY STRIPPING TURBULENCE, MASS TRANSFER AND CHEMICAL REACTION FROM THE MAIN REF 1 CODE
- IT IS EASY TO PUT ALL THE FEATURES INTO A SINGLE CODE.

MPF 4	3/15	SCOPE OF 'GENMIX 2P': UNSTEADY-STATE MODE (TRNSNT = .TRUE.)

- GENMIX 2P ALSO SOLVES ONE-DIMENSIONAL UNSTEADY DIPSERSED-FLOW AND GRAVITY-STRATIFIED-FLOW PROBLEMS.
- THE ALLOWANCE FOR VISCOUS, HEAT-TRANSFER AND MASS-TRANSFER EFFECTS IS AS FOR TRNSNT = .FALSE..
- GENMIX 2P EMPLOYS THE (NORMALISED) STREAM FUNCTION OF THE FIRST PHASE AS CROSS-STREAM VARIABLE, AS DESCRIBED IN LECTURE 3.
- FOR TRNSNT = .TRUE., THE VELOCITIES u_1 AND u_2 ARE SET EQUAL TO UNITY; THEN x HAS THE SIGNIFICANCE OF TIME. OTHERWISE THE ANALYSIS IS THE SAME.
- GENMIX 2P DIFFERS FROM "PLANT" (LECTURE 2) IN THAT:
  - PLANT OPERATES ONLY IN AN UNSTEADY-STATE MODE.
  - PLANT EMPLOYS A GRID FIXED IN SPACE, NOT RELATIVE TO MATERIAL.

MPF 4	4/15	STRUCTURE OF 'GENMIX 2P': LIST OF SUBROUTINES.

- GENMIX 2P IS STRUCTURED SIMILARLY TO THE PUBLISHED GENMIX; IT HAS TWO FURTHER SUBROUTINES: COMP2P and ADJ2P.
- THE COMPLETE LIST OF SUBROUTINES IS:
  - MAIN WHICH CALLS COMP WHICH CALLS PHYS AND WALL;
  - MAIN ALSO CALLS ADJ,
AND COMP2P AND ADJ2P,
AND OUTPUT WHICH CALLS PLOTS.
  - OUTPUT WHICH OVERWRITES SOME OF THE INFORMATION SUPPLIED IN BLOCK DATA.
- COMP (APART FROM ONE STATEMENT), PHYS, WALL AND PLOTS ARE IDENTICAL WITH THE CORRESPONDING SUBROUTINES IN GENMIX 0.

MPF 4	5/15	STRUCTURE OF 'GENMIX 2P': FUNCTIONS OF SUBROUTINES, 1.

SUBROUTINES WHICH THE USER WILL MODIFY

- BLOCK DATA AND INPUT RECEIVE PROBLEM-DEFINING INFORMATION.
- MAIN FIRST SETS UP THE GRID AND THE INITIAL CONDITIONS, AND THEN ORGANIZES THE MARCHING-INTEGRATION PROCESS.
- OUTPUT DETERMINES WHAT WILL BE PRINTED OUT BEFORE, DURING AND AFTER THIS PROCESS.

SUBROUTINES WHICH THE USER MAY MODIFY

- PHYS EMBODIES TRANSPORT-PROCESS AND SOURCE INFORMATION, E.G. INTERPHASE-TRANSPORT LAWS.
- WALL PROVIDES SPECIAL INFORMATION ABOUT TRANSFER PROCESSES WITH BOUNDARY WALLS.

MPF 4	6/15	STRUCTURE OF 'GENMIX 2P': FUNCTIONS OF SUBROUTINES, 2

SUBROUTINES WHICH THE USER NEED NOT MODIFY

- COMP AT ENTRY INIT PROVIDES INDICES AND DEFAULT VALUES.
COMP AT ENTRY GRID SETS UP THE NORMALISED STREAM-FUNCTION VALUES.
COMP AT ENTRY DISTAN COMPUTES y's FROM ρ_1's u_1's, ETC.
COMP AT ENTRY SOLVE SETS UP AND SOLVES FDE'S FOR u_1 AND h_1.
- COMP2P SETS UP AND SOLVES FDE'S FOR (v_2/u_2), u_2, h_2, r_2.
- ADJ2P COMPARES r_2'sWITH THOSE PRESUMED AT THE START OF THE ITERATION LOOP, AND ADJUSTS THEM BY A USER-SELECTED METHOD.
- PLOTS PROVIDES LINE-PRINTER-GENERATED GRAPHICAL OUTPUT.

MPF 4	7/15	STRUCTURE OF 'GENMIX 2P': ARRANGEMENT OF MAIN

- MAIN IS ARRANGED IN 12 CHAPTERS. THE FIRST 5 CONCERN PROBLEM SET-UP; THE LAST 7 COMPRISE THE MARCHING STEP.
- MAIN TAKES INFORMATION FROM BLOCK DATA, WHICH HAS A CORRESPONDING 12-CHAPTER FORMAT.
- CHAPTER 1 CONCERNS PRELIMINARIES (DECLARATIONS, NEEDED ARITHMETIC-STATEMENT FUNCTIONS, RUN ORGANIZATION).
- CHAPTER 2 CALLS INIT AND GRID, SETS UP THE GRID.
- CHAPTER 3 DETERMINES THE DEPENDENT VARIABLES TO BE SOLVED.
- CHAPTER 4 MANIPULATES MATERIAL-PROPERTY DATA.
- CHAPTER 5 PROVIDES INITIAL CONDITIONS FOR THE MARCHING INTEGRATION.

MPF 4	8/15	STRUCTURE OF 'GENMIX 2P': ARRANGEMENT OF MAIN, CONTINUED

- CHAPTER 6 STARTS THE FORWARD-STEPPING PROCESS BY DETERMINING STEP SIZE AND STORING UPSTREAM (EARLY-TIME) VALUES.
- CHAPTER 7 PROVIDES BOUNDARY CONDITIONS FOR THE STEP.
- CHAPTER 8 STARTS AN INTEGRATION LOOP, CALLS SOLVE TO PROVIDE END-OF-STEP VALUES OF u_1 AND $\dot{h}_1$.
- CHAPTER 9 UPDATES ρ_1 IF NECESSARY, CALLS DISTAN FOR y's, COMPUTES FURTHER CELL-GEOMETRY QUANTITIES INCLUDING v_1/u_1, OBTAINS p{y} BY INTEGRATING THE LATERAL-MOMENTUM EQUATION, CALLS COMP2P TO SOLVE FOR (v_2/u_2), u_2, h_2, r_2.
- CHAPTER 10 CALLS ADJ2P FOR ADJUSTMENTS, ENDS ITERATION LOOP.
- CHAPTER 11 CALLS OUTPUT.
- CHAPTER 12 DECIDES WHETHER TO MAKE ANOTHER STEP OR TERMINATE.

MPF 4	9/15	MAJOR VARIABLES OF 'GENMIX 2P': VARIABLES ASSOCIATED WITH THE GRID

- XU,XD: UPSTREAM & DOWNSTREAM VALUES OF x (DISTANCE OR TIME).
- PSII, PSIE,PEI: ψ_{1I}, ψ_{1E}, $\psi_{1E} - \psi_{1I}$, WHERE I & E DENOTE THE TWO EDGES OF THE FINITE-DIFFERENCE GRID.
- OM(I): $\omega_i \equiv (\psi_{1,i} - \psi_{1I})/(\psi_{1E} - \psi_{1I})$, THE NORMALISED STREAM FUNCTION.
- I: INDEX OF GRID POINTS, = 1 AT I BOUNDARY, = N AT E BOUNDARY.
- Y(I): y VALUE OF THE i'th GRID POINT.
- R(I): RADIUS (FOR AXI-SYMMETRICAL FLOWS) OF THE i'th GRID POINT.
- YINT(I): y VALUE FOR INTERFACE BETWEEN i'th AND i+1'th CELL.
- TEE(I): dy/dx FOR SAID INTERFACE.
- ADPEI(I): DOWNSTREAM AREA OF i'th CELL ÷ PEI.

MPF 4	10/15	MAJOR VARIABLES OF 'GENMIX 2P': DEPENDENT VARIABLES

- U(I): x-DIRECTION VELOCITY OF FIRST PHASE.
- H(I): SPECIFIC ENTHALPY OF FIRST PHASE.
- AL(I): v/u FOR FIRST PHASE AT INTERFACE BETWEEN i'th AND i+1'th CELL.
- U2(I): x-DIRECTION VELOCITY OF SECOND PHASE.
- H2(I): SPECIFIC ENTHALPY OF SECOND PHASE.
- AL2(I): v/u FOR SECOND PHASE AT i ~ i+1 INTERFACE.
- R2(I): VOLUME FRACTION OF SECOND PHASE.
- UU(I), HU(I), ALU(I), U2U(I), ETC: VALUES OF VARIABLES AT UPSTREAM FACE OF FORWARD STEP.

MPF 4 — $\frac{11}{15}$ — INPUT VIA BLOCK DATA OR SUBROUTINE INPUT (SELECTED ITEMS)

- CHAPTER 1: • PROBLEM TYPE IS SET VIA HEAT, TRNSNT, STRAT.
  - ITEST, JTEST, JTEST1: INDICES WHICH, APPROPRIATELY SET, ELICIT OUTPUT OF INTERMEDIATE QUANTITIES, EG COEFFICIENTS.
- CHAPTER 2: • N: NUMBER OF GRID POINTS ACROSS THE STREAM.
  - KRAD: INDEX = 1 FOR PLANE FLOW, = 2 FOR AXI-SYMMETRICAL FLOW.
  - XULAST: VALUE OF x FOR WHICH INTEGRATION SHOULD BE STOPPED.
  - LASTEP: VALUE OF STEP COUNTER, ISTEP, FOR STOPPING INTEGRATION.
- CHAPTER 3: • JH, JAL, JU2, ETC: VALUE OF INDICES BY MEANS OF WHICH H(I) CAN BE REFERRED TO AS F(I,JH), ETC.

MPF 4 — $\frac{12}{15}$ — INPUT VIA BLOCK DATA OR SUBROUTINE INPUT (FURTHER SELECTED ITEMS)

- CHAPTER 4: • RHO1, RHO2: DENSITIES OF PHASES 1 AND 2.
  - CP1, CP2: CONSTANT-PRESSURE SPECIFIC HEATS OF THE PHASES.
  - FIP, HIP: INTERPHASE COEFFICIENTS OF FRICTION AND HEAT TRANSFER.
- CHAPTER 5: • U/20*..., H/20*..., ETC: INITIAL VALUES OF VARIABLES ON STARTING LINE OF MARCHING INTEGRATION. HERE UNIFORM VALUES ARE IMPLIED; BUT ANY VALUES CAN BE INPUT.
  - PSII, PSIE: VALUES WHICH, TOGETHER WITH U, RHO1, ETC WILL DEFINE THE INITIAL WIDTH, Y(N), OF THE GRID.
  - RMI, RME: INITIAL VALUES OF RATES OF PHASE-1 MASS TRANSFER ACROSS THE I AND E BOUNDARIES IN DIRECTION OF POSITIVE y.

MPF 4 — $\frac{13}{15}$ — INPUT VIA BLOCK DATA (SELECTED ITEMS, CONTINUED)

- CHAPTER 6: • FRA: FORWARD STEP, DX, ÷ Y(N).
- CHAPTER 7: • VALUES OF DEPENDENT VARIABLES FOR I AND E BOUNDARIES MAY BE SUPPLIED HERE, IF MATCHING STATEMENTS ARE INSERTED IN CHAPTER 7 OF MAIN.
- CHAPTER 8: • NITER: MAXIMUM NUMBER OF ITERATIONS OF A SINGLE FORWARD-STEP CALCULATION.
- CHAPTER 9: • TANI: VALUE OF $(v_1/u_1)_I$.
- CHAPTER 10: • METHOD: THE r_1 - CORRECTION METHOD (LECTURE 3).
- CHAPTER 11: • NSTAT: FREQUENCY OF PRINT-OUT OF "STATION VARIABLES".
  - NPROF, NPLOT: DITTO FOR PROFILE TABLES, AND PLOTS, RESPECTIVELY.

MPF 4 — $\frac{14}{15}$ — OUTPUT

- KINDS OF OUTPUT: • HEADINGS. • STATION VARIABLES.
  - PROFILE TABLES. • CROSS-STREAM (PROFILE) PLOTS.
  - LONGITUDINAL PLOTS.
- HEADINGS: • PRINTED AT THE START OF INTEGRATION (ISTEP=0) FROM CHAPTER B OF OUTPUT.
  - THEY COMPRISE PROBLEM-DEFINING INFORMATION.
- STATION VARIABLES: • NON-ARRAYED VARIABLES, VALID FOR A STEP, AND PRINTED FROM CHAPTER D OF OUTPUT.
- PROFILE TABLES: • ARRAYED VARIABLES, VALID FOR A STEP, AND PRINTED FROM CHAPTER E OF OUTPUT.
- CROSS-STREAM PLOTS: • PLOTS OF PROFILES FOR A STEP.
- LONGITUDINAL PLOTS: • STATION VARIABLES, PLOTTED VERSUS x AT THE END OF INTEGRATION.

MPF 4	15/15	AN EXAMPLE

- PROCESS: 2D PLANE STEADY DISPERSED FLOW, WITHOUT LONGITUDINAL PRESSURE GRADIENT, AND DENSE PHASE FALLING UNDER GRAVITY.
- DATA:
  - INITIAL VALUES: $u_1 = 1.0$, $u_2 = 2.0$, $R_2 = 0.1$, $Y_N = 0.1$
  - PROPERTIES: $\rho_1 = 1.0$, $\rho_2 = 10$; FIP = 0.1
  - OTHER DATA: METHOD = 3, LASTEP = 100, NITER = 5, & SEE LISTING.
- RESULTS: SEE SUPPLIED OUTPUT
- OBSERVATIONS:
  - INTERPHASE FRICTION CAUSES u_1 TO INCREASE, u_2 TO DECREASE
  - R_2 INCREASES AT BOTTOM OF DUCT, DECREASES AT TOP, BECAUSE OF GRAVITY.
  - CORRESPONDINGLY, v_1/u_1 IS POSITIVE, v_2/u_2 NEGATIVE.

MPF 5	1/15	LECTURE 5: ONE-DIMENSIONAL UNSTEADY 2-PHASE STRATIFIED FLOW, WITHOUT PHASE CHANGE

CONTENTS

NOTE: EMPHASIS WILL BE ON THE DIFFERENCES BETWEEN DISPERSED AND STRATIFIED FLOW.

MPF 5	2/15	EXEMPLIFICATION OF SCOPE OF LECTURE

- OCCURRENCE:
  - IN GAS PIPE LINES, EG UNDER THE OCEAN, WHERE CONDENSATE ALSO FLOWS.
  - IN CONDENSERS AND BOILERS WHERE THE TUBES ARE NEARLY HORIZONTAL AND THE PHASE CHANGE IS INSIDE THE TUBE. FLOW VELOCITIES MUST BE LOW.
- SPECIAL FEATURE:
  - INCLINATIONS IN THE HEIGHT OF INTERFACE TEND TO BE REDUCED TO ZERO BY GRAVITATIONAL ACTION.
  - THIS IS EXPRESSED BY INTRODUCING A NEW TERM IN THE MOMENTUM EQUATION.

MPF 5	3/15	PARTIAL DIFFERENTIAL EQUATIONS

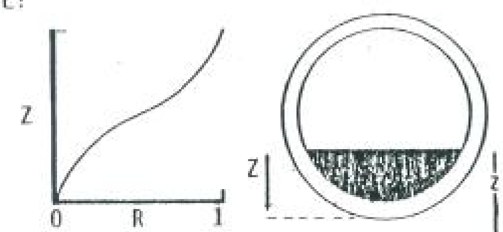
- CONTINUITY: AS FOR DISPERSED FLOW.
- MOMENTUM: $\frac{\partial}{\partial t}(\frac{ru}{v}) + \frac{\partial}{\partial x}(\frac{ru}{v}u) = -uf_s + (U-u)f_I - r\frac{\partial p}{\partial x}$

  AND: $\frac{\partial}{\partial t}(\frac{RU}{V}) + \frac{\partial}{\partial x}(\frac{RU}{V}U) = -UF_s + (u-U)F_I - R\frac{\partial p}{\partial x} - g(\frac{1}{V} - \frac{1}{v})\frac{\partial z}{\partial x}$
- NOMENCLATURE: $z \equiv$ INTERFACE HEIGHT ABOVE A HORIZONTAL BASE.
  $p \equiv$ "REDUCED PRESSURE" IN LIGHT PHASE, IE, PRESSURE + (g/v) ELEVATION.
- REMARKS:
  - $-g(V^{-1} - v^{-1})\partial z/\partial x$ REPRESENTS THE TERM TENDING TO MAKE z INDEPENDENT OF x.
  - f_I AND F_I ARE RELATED BY:
    $f_I \quad = F_I$.

MPF 5	4/15	AUXILIARY RELATIONS

- INTERFACE ELEVATION: $z = z(x, R)$ IN GENERAL.
- WHEN PIPE DIAMETER IS UNIFORM: $z = z_b(x) + Z(R)$

  WHERE z_b REPRESENTS ELEVATION OF BOTTOM OF INSIDE OF PIPE.
- FOR A CIRCULAR PIPE:



MPF 5	5/15	FINITE- DOMAIN EQUATIONS, 1: THE UNCHANGED EQUATIONS

- CONTINUITY: AS FOR DISPERSED FLOWS; COMPACT FORM IS:

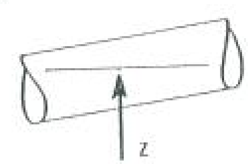
  $r_P = a_W r_W + a_E r_E + a_{P-} r_{P-}$

  $R_P = A_W R_W + A_E R_E + A_{P-} R_{P-}$
- MOMENTUM: FOR THE LIGHT PHASE, AS FOR DISPERSED FLOW, WITHOUT GRAVITY TERM:

  $u_w = a_{ww}u_{ww} + a_e u_e + a_{w-}u_{w-} + b_w(p_W - p_P) + c_w u_w$
- NOTES:
  - THE MEANINGS OF a's, A's ETC, CAN BE DEDUCED FROM INSPECTION OF PANELS 2.5 AND 2.6.
    THEY DIFFER FROM THOSE OF LECTURE 1.

MPF 5	6/15	FINITE- DOMAIN EQUATIONS, 2: MOMENTUM FOR THE HEAVY FLUID

- EQUATION: $U_w = A_{ww}U_{ww} + A_e U_e + B_w(p_W - p_P) + C_w U_w + D_w(z_W - z_P)$
- NOTES:
  - THE LAST TERM IS THE NEW ONE.
  - BECAUSE $z = z(R, x)$, IT IS POSSIBLE TO EXPRESS THE LAST TERM AS:
    $E_w(R_W - R_P) + G_w$
    BY LINEARIZATION.



  - AS A RULE, E_w AND G_w THEMSELVES DEPEND ON x AND R.

MPF 5	7/15	FINITE- DOMAIN EQUATIONS, 3: DISCUSSION OF THE PROBLEM OF SOLUTION

- THE "NEW TERM" IS OFTEN A MUCH BIGGER INFLUENCE ON HEAVY-FLUID MOTION THAN THE PRESSURE TERM, ESPECIALLY WHEN DENSITIES DIFFER GREATLY.
- WHEN $R \ll 1$, $V \ll v$, AND INTERPHASE FRICTION IS SMALL, THE HEAVY PHASE IS SCARCELY INFLUENCED BY THE PRESENCE OF THE LIGHT ONE; FOR ITS PRESSURE VARIATIONS ARE SMALL.
- CONSEQUENTLY THE CONTINUITY ERRORS ARE BEST ADJUSTED BY CORRECTING R, RATHER THAN P; OR BOTH AT ONCE.
- THE NEW TERM HAS BROUGHT A NEW LINK BETWEEN U AND R.

MPF 5	8/15	"IPSA" FOR GRAVITY-STRATIFIED FLOW: VERSION 1

- NATURE: THE ADDITIONAL TERM IS INTRODUCED INTO THE MOMENTUM EQUATION FOR U_*; BUT NO OTHER CHANGES ARE MADE.
- RESULTS: PROVIDED THE DENSITY OF THE LIGHT FLUID IS NOT TOO FAR BELOW THAT OF THE HEAVY ONE, CONVERGENCE IS OBTAINED, ALBEIT MORE SLOWLY.
- REMARKS: THIS METHOD CAN BE SUCCESSFUL EVEN WITH $v/V = 1000.$; BUT IT IS BOUND TO FAIL WHEN v/V IS LARGE ENOUGH, BECAUSE PRESSURE DIFFERENCES ARE TOO EASILY EQUALISED BY MOTION OF THE LIGHT PHASE.

MPF 5	9/15	"IPSA" FOR GRAVITY-STRATIFIED FLOW: VERSION 2

- NATURE: AFTER CALCULATION OF u_* AND U_*, AN R-CORRECTION EQUATION IS SET UP AND SOLVED. THE PRESSURE-CORRECTION EQUATION IS SOLVED ONLY AFTERWARDS, IF AT ALL.
- DETAILS:
  - U' IS CONNECTED WITH R' BY DIFFERENTIATION OF THE EQUATION OF SLIDE 6:

    e.g. $U'_W = D_W \left(\frac{\partial z_W}{\partial R_W} R'_W - \frac{\partial z_P}{\partial R_P} R'_P\right)$
  - THE R EQUATION IS WRITTEN IN CORRECTION FORM:

    $R_P\,([-U_W]' + [U_e]') + (V_P/V_{P-})(\delta x/\delta t)\,R_P'$

    $= R_W\,[U_W]_* + R_E\,[-U_e]_* + R_{P-}\,(\delta x/\delta t)$
  - U''s ARE ELIMINATED; R''s ARE OBTAINED BY TDMA.

MPF 5	10/15	"IPSA" FOR GRAVITY-STRATIFIED FLOW: VERSION 3

- NATURE: THE VARIATION OF U WITH R IS TAKEN INTO ACCOUNT IMPLICITLY IN THE R EQUATION.
- DETAILS:
  - THE R EQUATION IS WRITTEN IN CORRECTION FORM AS IN PANEL 9, BUT WITH:

    $R'_P\,([-U_W]_* + [U_e]_*) - R'_W[U_W] - R_E[-U_e]_*$,

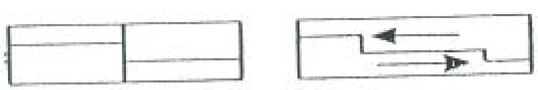
    ADDED TO THE LEFT-HAND SIDE.
- THE SUBSTITUTIONS OF PANEL 5.10 ARE MADE FOR U_W', U_e'.
- THE EQUATIONS ARE SOLVED FOR R' USUALLY POINT-BY-POINT.
- THE ASSOCIATED VALUES OF U_W' AND U_e' ARE COMPUTED.
- THE VALUES OF R AND U ARE INCREMENTED.
- THE MOMENTUM AND PRESSURE-CORRECTION EQUATIONS ARE THEN SOLVED.

MPF 5	11/15	"IPSA" FOR GRAVITY-STRATIFIED FLOW DISCUSSION

- MOST EXPERIENCE HAS BEEN GAINED SO FAR WITH VERSION 1. IT IS BUILT INTO GENMIX 2P.
- ALL VERSIONS MUST PRODUCE THE SAME SOLUTION, IF THE ITERATION NUMBER IS LARGE, WHEN CONVERGENCE IS OBTAINED AT ALL.
- VERSIONS 2 AND 3 GIVE CONVERGENCE WITH FEWER ITERATIONS AT LARGER TIME STEPS THAN VERSION 1.
- VERSION 3 IS SLIGHTLY SUPERIOR TO VERSION 2 IN MOST CASES.
- IN GENERAL, IT MUST BE RECOGNIZED, MORE THAN ONE SOLUTION PROCEDURE EXISTS FOR THE SAME EQUATION SET.
- FOR EXAMPLE, THE r AND R EQUATIONS MAY BE DIVIDED BY v AND V RESPECTIVELY BEFORE ADDITION, THEN THE MASS CONTINUITY EQUATION IS USED FOR p', NOT THE VOLUMETRIC ONE.

MPF 5	12/15	PARTICULAR EXAMPLE, 1: PROBLEM STATEMENT

DETERMINE WHAT ENSUES WHEN A DIAPHRAGM IS REMOVED FROM A TUBE IN WHICH IT HAD MAINTAINED DIFFERENCES IN LIQUID LEVEL.

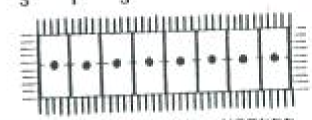
BEFORE:

: AFTER

NOTES: THIS IS IN SOME RESPECTS THE EQUIVALENT OF THE SHOCK-TUBE PROBLEM.

THE EXPERIMENT IS EASY TO PERFORM; AND THE TUBE MAY BE TILTED, BENT, SQUEEZED, ETC., TO PROVIDE VARIETY.

MPF 5	13/15	PARTICULAR EXAMPLE, 2: DETAILS OF CALCULATION

- COMPUTER CODE: PLANT (STRATIFIED-FLOW OPTION).
- INITIAL CONDITIONS: $u = U = 0$; $R = R_{left}$ or $R = R_{right}$ ACCORDING TO POSITION; p IS IMMATERIAL FOR INCOMPRESSIBLE FLUIDS.
- BOUNDARY CONDITIONS: $u = U = 0$ AT EACH END.
- AUXILIARY RELATIONS: f_l, f_s, F_l, F_s, PIPE CROSS-SECTION.
- GRID ARRANGEMENT:
  - • FOR p, r, R;
  - – FOR u, U.



- FOR GOOD ACCURACY, 50 OR MORE NODES MAY BE NEEDED.
- TIME STEP: NOT GREATER THAN: $\delta x/\sqrt{\frac{1}{2}g(z_{left}+z_{right})}$

MPF 5	14/15	PARTICULAR EXAMPLE, 3: DISCUSSION

- RESULTS: THESE HAVE BEEN PRESENTED BY BAGHDADI ET AL.
- ACCURACY: AGREEMENT IS GOOD WITH ANALYTICAL SOLUTION FOR WAVE SPEED (KNOWN FOR ZERO-FRICTION CASE).
- OTHER EXAMPLES STUDIED INCLUDE:
  - SUDDEN TILTING OF TUBE.
  - SUDDEN OPENING OF PIPE TO A RESERVOIR (NB OUTLET BOUNDARY CONDITION IS R = MAXIMUM).
  - STEADY INFLOW OF LIQUID AND GAS INTO BENT PIPE, LEADING TO UNSTEADY OUTFLOW.

MPF 5	15/15	CONCLUDING REMARKS

- THE ONE-DIMENSIONAL TWO-PHASE STRATIFIED-FLOW MODEL REDUCES TO THAT USED IN "SHALLOW-WATER THEORY" BY CIVIL ENGINEERS, WHEN R IS SMALL.
- MANY PRACTICALLY-OCCURRING PHENOMENA CAN BE ANALYSED WITH THE AID OF THE MODEL; AND MANY SIMPLE LABORATORY EXPERIMENTS CAN BE DEVISED AS TESTS.
- SO FAR, FEW EXPERIMENTAL STUDIES HAVE BEEN MADE.
- REALISTIC RESULTS ARE TO BE EXPECTED, BECAUSE FRICTION COEFFICIENTS ARE EASIER TO ESTIMATE THAN FOR DISPERSED FLOW.

MPF 6	1/15	LECTURE 6: ONE-DIMENSIONAL TWO-PHASE FLOW WITH INTERPHASE MASS TRANSFER

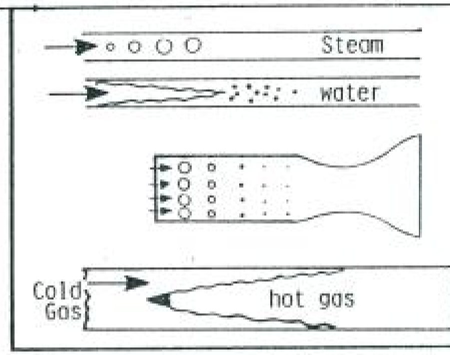
CONTENTS

- PRACTICAL RELEVANCE
- DIFFERENTIAL EQUATIONS FOR STEAM-WATER SYSTEM
- AUXILIARY RELATIONS
- A STEAM-GENERATOR EXAMPLE
- DISCUSSION

NOTE: IN MANY CASES THE INTERPHASE MASS TRANSFER IS ASSOCIATED WITH, AND MAY BE THE RESULT OF, HEAT TRANSFER; BUT IN THE PRESENT LECTURE, THAT ASPECT IS NOT EMPHASISED.

MPF 6	2/15	PRACTICAL RELEVANCE

- STEAM BOILERS:
- STEAM CONDENSERS:
- LOCA (LOSS-OF-COOLANT ACCIDENT):
- LIQUID-PROPELLANT ROCKET MOTORS:
- AFTER-BURNER SYSTEMS OF GAS TURBINES:



- NOTES:
  - IN EACH CASE THE 1D CHARACTER IS FICTIONAL.
  - BOTH STEADY AND UNSTEADY FLOWS ARISE.

MPF 6	$\frac{3}{15}$	DIFFERENTIAL EQUATIONS FOR THE STEAM-WATER SYSTEM, 1: CONTINUITY AND MOMENTUM

- CONTINUITY: $\frac{\partial r}{\partial t} + \frac{\partial}{\partial x}(ru) = \frac{r}{v}\frac{Dv}{Dt} + \dot{v}$

  $\frac{\partial R}{\partial t} + \frac{\partial}{\partial x}(RU) = \frac{R}{V}\frac{DV}{Dt} + \dot{V}$
- MOMENTUM: AS FOR LECTURES 2 AND 3, BUT WITH $[\dot{v}/v]$ AND $[-\dot{V}/V]$ ADDED TO f_I AND F_I RESPECTIVELY.
- NOMENCLATURE: $\dot{v}$, $\dot{V}$ ≡ VOLUMES OF STEAM AND WATER CREATED PER UNIT VOLUME AND TIME.
- DISCUSSION: $\dot{v}$ AND $\dot{V}$ VARY WITH x AND t AS A RESULT OF HEAT INPUT AND PRESSURE CHANGE.

  v (BUT SCARCELY V) VARIES WITH PRESSURE.

MPF 6	$\frac{4}{15}$	DIFFERENTIAL EQUATIONS, 2: THE ENERGY EQUATION

- RESTRICTION: THERMAL EQUILIBRIUM BETWEEN PHASES WILL BE POSTULATED; KINETIC ENERGY IS NEGLECTED.
- DEFINITIONS: h, H ≡ SPECIFIC ENTHALPIES

  $\dot{q}$ ≡ TOTAL HEAT INPUT PER UNIT VOLUME.
- EQUATION: $$\frac{\partial}{\partial t}\left(\frac{rh}{v} + \frac{RH}{V} - p\right) + \frac{\partial}{\partial x}\left(\frac{rh}{v}u + \frac{RH}{V}U\right) = \dot{q}$$
- DISCUSSION: THIS HOLDS IN SINGLE-PHASE REGIONS (IE r = 0 OR r = 1) AS WELL AS MIXED-PHASE REGION.

  IN MIXED-PHASE REGION, h, v, H, V HAVE THEIR SATURATION VALUES.

  IT IS IMMATERIAL INTO WHICH PHASE THE HEAT GOES.

MPF 6	$\frac{5}{15}$	DIFFERENTIAL EQUATIONS, 3: RELATION OF ENERGY TO CONTINUITY EQUATIONS

- ALGEBRAIC MANIPULATION: SUBTRACT FROM ENERGY EQUATION, h/v TIMES r-EQUATION AND H/V TIMES R-EQUATION.
- RESULT: $$\dot{m} = \left(\dot{q} + \frac{\partial p}{\partial t} - \frac{r}{v}\frac{Dh}{Dt} - \frac{R}{V}\frac{DH}{Dt}\right) / (h - H)$$
- DISCUSSION:
  - $\dot{m} \equiv \dot{v}/v \equiv -\dot{V}/V$
  - WHERE THIS EQUATION IS USEFUL, IE:

    WHERE $\dot{m} \neq 0$, THEN $(h - H) = h_{sat} - H_{sat} \equiv \lambda$, THE LATENT HEAT.
  - WHERE THE PRESSURE IS CONSTANT, THIS REDUCES TO: $\dot{m} = \dot{q}/\lambda$.

MPF 6	$\frac{6}{15}$	AUXILIARY RELATIONS

- THERMODYNAMICS:
  - FOR SATURATION CONDITIONS (IE $0 < r < 1$), h, H, v, V, λ, T ARE KNOWN FUNCTIONS OF PRESSURE.
  - ELSEWHERE, h AND H DEPEND ON p AND T.
- HEAT FLUX:
  - SOMETIMES, $\dot{q}$ (x,t) IS PRESCRIBED, EG IN A NUCLEAR REACTOR.
  - MORE OFTEN, $\dot{q}$ (x,t) DEPENDS ON LOCAL u, U, r, R, T.
- "HEAT CONDUCTION": (REMINDER)
  - x-DIRECTION DIFFUSION-LIKE HEAT TRANSFER IS BEING NEGLECTED.

    IT CAN BE SIMULATED BY REDISTRIBUTION OF HEAT FLUXES.

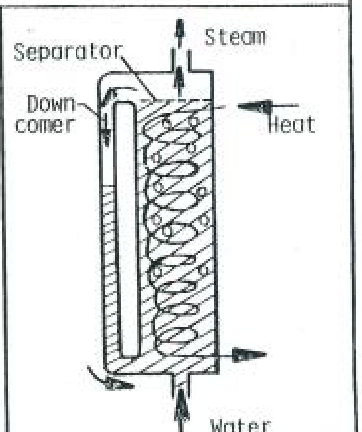
MPF 6	7/15	APPRECIATION OF THE MATHEMATICAL PROBLEM

- IN THE MIXED-PHASE REGION THERE IS NO NEED TO SOLVE THE ENERGY EQUATION, MERELY TO USE IT SO AS TO OBTAIN $\dot{v}$ AND $\dot{V}$ FOR THE r AND R EQUATIONS.
- IN THE SINGLE-PHASE REGION, THE ENERGY EQUATION MUST BE SOLVED, SO THAT THE BOUNDARIES OF THE REGION CAN BE DETERMINED.
- THESE BOUNDARIES WILL NOT NORMALLY COINCIDE WITH FINITE-DOMAIN BOUNDARIES; SO PROGRAMMING A TWO-PART SOLUTION PROCEDURE IS NOT STRAIGHTFORWARD (SEE NEXT LECTURE).
- ONCE THE $\dot{m}$ DISTRIBUTION HAS BEEN DETERMINED, THE SOLUTION PROCEDURE IS AS BEFORE.

MPF 6	8/15	A PARTICULAR PROBLEM: DESCRIPTION

- THE PROBLEM:

  A STEAM-WATER MIXTURE, FLOWING IN A LOOP, EXPERIENCES A SUDDEN INCREASE IN HEAT FLUX.
- PRESUMPTIONS:
  - 1D ANALYSIS SUFFICES.
  - INTERPHASE FRICTION IS FINITE.
  - GRAVITY IS INFLUENTIAL.
  - SEPARATORS AT TOP ARE 99% EFFICIENT.



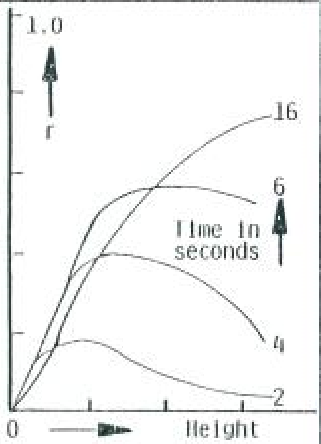
MPF 6	9/15	THE STEAM-GENERATOR PROBLEM: INITIAL AND BOUNDARY CONDITIONS

- INITIAL CONDITIONS:
  - VOLUME FRACTION: r = 0.01, R = 0.99.
  - VELOCITIES: u = U = 0.519 m/s
  - DOWNCOMER LEVEL: 12.0 m (NB GENERATOR HEIGHT = 13.0 m).
  - ENTHALPY: APPROPRIATE TO SATURATION.
  - HEAT INPUT: ZERO AT START, SUDDENLY INCREASED.
- NOTES:
  - THE DOWNCOMER HEIGHT IS HELD CONSTANT FOR FIRST 6.0s BY ADDITION OR WITHDRAWAL OF FLUID; IT IS THEN ALLOWED TO ADJUST ITSELF.
  - THESE CONDITIONS ARE PURELY ILLUSTRATIVE; ANY CAN BE POSTULATED.
  - THE STEAM-DOME PRESSURE IS HELD CONSTANT.

MPF 6	10/16	THE STEAM-GENERATOR PROBLEM: SOME DETAILS

- CODE USED: PLANT, WITH ADDITIONS TO ALLOW FOR DOWNCOMER BEHAVIOUR.
- GRID USED: 34 PRESSURE NODES, 33 VELOCITY NODES.
- TIME STEP: 0.2 SECONDS.
- ITERATION NUMBER PER TIME STEP: 10
- INTERPHASE FRICTION LAW: LINEAR, VARIOUS VALUES HAVE BEEN USED FOR LOWER- AND UPPER-LIMIT STUDIES. THE ONE USED FOR THE ILLUSTRATION IS "MODERATE".
- COMPUTER TIME: 4 SECONDS ON CDC 7600 FOR 100 TIME STEPS.
- CONVERGENCE: GOOD.

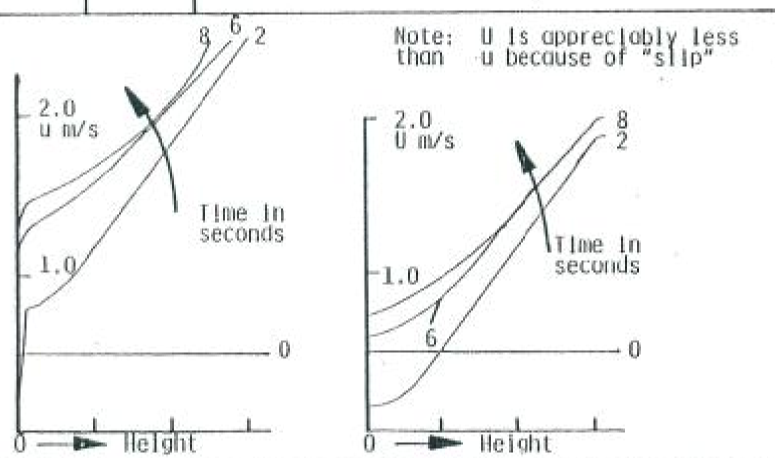
MPF 6	11/15	VARIATION OF STEAM VOLUME FRACTION r WITH POSITION AND TIME



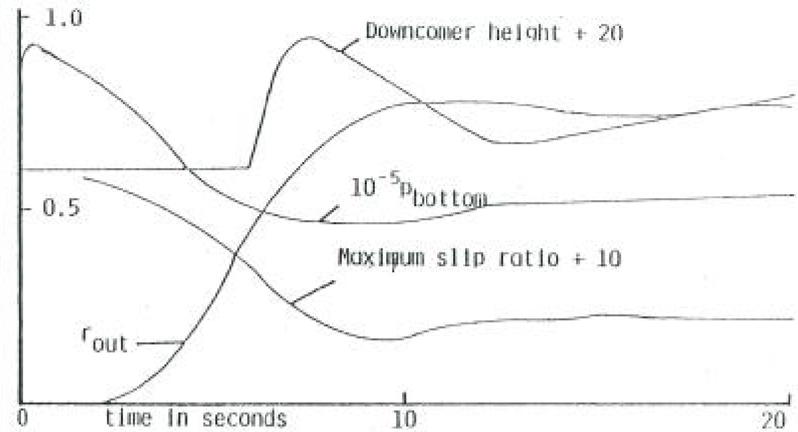
NOTES:

- STEAM GENERATED IN THE CENTRAL PART OF THE DUCT MUST FIRST DISPLACE LIQUID RESIDING THERE.
- THIS FLOWS UPWARD IN EXAMPLE CONSIDERED; BUT, IN SOME CONDITIONS, IT CAN BE FORCED BACK INTO THE DOWNCOMER.

MPF 6	12/15	VARIATIONS OF STEAM AND LIQUID VELOCITY, u AND U, WITH POSITION AND TIME



MPF 6	13/15	VARIATIONS OF VARIOUS QUANTITIES WITH TIME



MPF 6	14/15	OBSERVATIONS DERIVED FROM OTHER COMPUTATIONS OF THE SAME TYPE

- INCREASING INTERPHASE FRICTION FACTOR LEADS FINALLY TO HOMOGENEOUS-FLOW SOLUTION.
- DECREASING THIS FACTOR INCREASES SLIP AND LOWERS r.
- IF SMALL TIME INTERVALS ARE USED, PRESSURE-WAVE EFFECTS IN VAPOUR ARE DETECTABLE.
- WALL FRICTION, AREA VARIATION ETC., WHEN INTRODUCED, PRODUCE THE EXPECTED EFFECTS.
- SUB-COOLING, SUPERHEATING, UNEQUAL TEMPERATURES, FINITE PHASE-CHANGE KINETICS, CAN ALL BE HANDLED (SEE NEXT LECTURE)
- SIGN CHANGES, AND SPATIAL VARIATIONS, OF HEAT FLUX BRING THE EXPECTED EFFECTS.
- "FALSE DIFFUSION" IS MAIN SOURCE OF INACCURACY.

MPF 6	15/15	CONCLUDING REMARKS

- "IPSA HANDLES PROBLEMS WITH MASS TRANSFER JUST AS EASILY AS THOSE WITHOUT.
- THE ROCKET-PROPELLANT PROBLEM CAN BE SOLVED BY THE SAME CODE, PROVIDED ONLY THAT THE VAPORISATION-RATE LAW IS PROVIDED.
- THE AFTER-BURNER PROBLEM CAN BE SOLVED BY THE SAME CODE, PROVIDED THAT THE ENTRAINMENT-RATE LAW IS GIVEN
- IN BOTH CASES, STEADY, TRANSIENT AND OSCILLATORY COMBUSTION PROCESSES CAN ALL BE MODELLED.
- INTERACTION BETWEEN BURNING RATES AND RELATIVE VELOCITY CAN BE SHOWN TO PROMOTE COMBUSTION INSTABILITY.

MPF 7	1/15	LECTURE 7: ONE-DIMENSIONAL TWO-PHASE FLOW WITH HEAT TRANSFER

CONTENTS

- THE PROBLEM
- PRACTICAL RELEVANCE
- DIFFERENTIAL EQUATIONS
- INTERPHASE TRANSFER RELATIONS
- FINITE-DOMAIN EQUATIONS
- SOLUTION PROCEDURE
- DISCUSSION

NOTE: THIS LECTURE EMPHASISES WHAT LECTURE 6 ONLY TOUCHED ON, VIZ THE ENERGY EQUATION(S).

MPF 7	2/15	THE PROBLEM UNDER CONSIDERATION

- THE TWO PHASES MAY HAVE DIFFERENT TEMPERATURES AT A GIVEN CROSS-SECTION.
- IF MASS TRANSFER IS POSSIBLE, ITS RATE IS OFTEN DETERMINED PARTLY BY THE HEAT TRANSFERS BETWEEN THE TWO PHASES AND THE INTERFACE, AND BY THE "LATENT HEAT".
- THERE MAY ALSO BE A "KINETIC" RESISTANCE TO PHASE CHANGE.
- IT IS NECESSARY, IN GENERAL, TO SOLVE ENERGY EQUATIONS FOR EACH PHASE, LINKED WITH THE MOMENTUM AND CONTINUITY EQUATIONS BY ALGEBRAIC RELATIONS.
- THE TASK HERE IS TO EXTEND "IPSA" SO AS TO DO THIS.

MPF 7	3/15	PRACTICAL RELEVANCE: SOME EXAMPLES

- WHEN COAL PARTICLES IGNITE AND BURN, THE BURNING RATE DEPENDS ON THE INTERFACE TEMPERATURE IN A NON-LINEAR FASHION; AND THIS DEPENDS ON THE HEAT-FLUX BALANCE.
- IN STEAM CONDENSERS, THE LIQUID (CONDENSATE) AND VAPOUR (STEAM WITH SOME CONTAMINATING AIR) ARE AT DIFFERENT TEMPERATURES.
- IN FLUIDISED-BED REACTORS, CHEMICALLY INERT PARTICLES DO NOT CHANGE PHASE; BUT THE HEAT TRANSFERS FROM GAS TO PARTICLES AND FROM PARTICLES TO COOLANT TUBES REQUIRE QUANTITATIVE MODELLING.
- IN LOSS-OF-COOLANT ACCIDENTS, AND RELATED FAST TRANSIENTS, SIGNIFICANT DEPARTURES FROM THERMAL EQUILIBRIUM CAN ARISE.

MPF 7	4/15	DIFFERENTIAL EQUATIONS FOR NEGLIGIBLE CHEMICAL REACTION AND KINETIC ENERGY (FOR SIMPLICITY)

- ENERGY EQUATION FOR LIGHT PHASE: $\frac{\partial}{\partial t}(\frac{rh}{V} - rp) + \frac{\partial}{\partial x}(\frac{rhu}{V}) = \dot{q}_0 + \dot{q}_1 + [\frac{\dot{v}}{v}]\, h_1$
- ENERGY EQUATION FOR HEAVY PHASE: $\frac{\partial}{\partial t}(\frac{RH}{V} - Rp) + \frac{\partial}{\partial x}(\frac{RHU}{V}) = \dot{Q}_0 + \dot{Q}_1 + [\frac{\dot{V}}{V}]\, H_1$
- NOMENCLATURE:
  - h, H ≡ SPECIFIC ENTHALPIES.
  - $\dot{q}_0, \dot{Q}_0$ ≡ HEAT FLUXES INTO PHASES FROM OUTSIDE.
  - $\dot{q}_1, \dot{Q}_1$ ≡ HEAT FLUXES INTO PHASES FROM INTERFACE.
- NOTES:
  - $\dot{v}/v = -\dot{V}/V$
  - [....] ≡ AMAX1 (0.0, ...)
  - $h_1 - H_1 = \lambda$, THE LATENT HEAT.

MPF 7	5/15	INTERPHASE-TRANSFER RELATIONS

- HEAT TRANSFERS FROM OUTSIDE: $\dot{q}_0 = a_0(h_0 - h)$; $\dot{Q}_0 = A_0(H_0 - H)$

  NOTE THAT a_0 AND A_0 MAY DEPEND ON r, u, R, U, ETC.
- HEAT TRANSFERS FROM INTERFACE: $\dot{q}_1 = a_1(h_1 - h)$; $\dot{Q}_1 = A_1(H_1 - H)$

  NOTE THAT a_1 AND A_1 MAY DEPEND ON ALL VARIABLES; $|u - U|$ WILL BE ESPECIALLY IMPORTANT.
- MASS TRANSFER: $\dot{m}\ (\equiv \dot{v}/v \equiv -\dot{V}/V) = -(\dot{q}_1 + \dot{Q}_1)/(h_1 - H_1)$

  AND ALSO: $\dot{m} = b_1(h_1 - h_{eq}) = B_1(H_1 - H_{eq})$
- NOTES:
  - a_0, a_1, A_0, A_1, ARE HEAT-TRANSFER COEFFICIENTS.
  - b_1 AND B_1 ARE KINETIC CONSTANTS, RELATED BY $b_1 c = B_1 C$, WHERE c AND C ARE THE SPECIFIC HEATS OF THE TWO PHASES.

MPF 7	6/15	DETERMINATION OF THE INTERFACE TEMPERATURE AND OF THE MASS-TRANSFER RATE

- INTRODUCTION OF TEMPERATURES: LET τ, T ≡ PHASE TEMPERATURES.

  SO: $h_1 - h = c(\tau_1 - \tau)$, $H_1 - H = C(T_1 - T)$, $h_1 - H = \lambda_1$,

  ALSO: $(h_1 - h_{eq})/c = (H_1 - H_{eq})/C = \tau_1 - \tau_{eq}$;

  AND: $\tau_1 = T_1$, $\tau_{eq} = T_{eq}(p)$.
- ELIMINATION OF $\dot{q}_1$, $\dot{Q}_1$, $\dot{m}$, AND PUTTING $\beta = b_1 c = B_1 C$,

  YIELDS: $$\tau_1 = T_1 = \frac{\beta\lambda_1 T_{eq} + a_1 c\tau + A_1 CT}{\beta\lambda_1 + a_1 c + A_1 C}$$
- REINTRODUCTION OF $\dot{m}$ NOW YIELDS:

$$\dot{m} = \frac{a_1 c(\tau - \tau_{eq}) + A_1 C(T - T_{eq})}{\lambda_1 + (a_1 c + A_1 C)/\beta}$$

MPF 7	$\frac{7}{15}$	INTERPHASE TRANSFER IN EXTREME CASES

- $\beta = 0$ (IE INFINITE KINETIC RESISTANCE TO PHASE CHANGE):
  $\dot{m} = 0$; AND $\tau_I = T_I = (a_i c\tau + A_I CT)/(a_i c + A_I C)$;
  $\dot{q}_I = -\dot{Q}_I = (T - \tau)/\{1/(a_i c) + 1/(A_I C)\}$
- $\beta = \infty$ (IE ZERO KINETIC RESISTANCE TO PHASE CHANGE):
  $\dot{m} = \{a_i c(\tau - T_{eq}) + A_I C(T - T_{eq})\}/\lambda$;
  $\dot{q}_I = (T_{eq} - \tau)\, a_i c$; $\dot{Q}_I = (T_{eq} - T)\, A_I C$.
- $\beta = \infty$, $a_I = \infty$, $A_I = \infty$ (IE ZERO KINETIC AND HEAT-TRANSFER RESISTANCES).
  SINCE $\dot{m}$, $\dot{q}_I$ AND $\dot{Q}_I$ MUST BE FINITE, $\tau = T = T_{eq}$.
- $\beta = \infty$, $A_I C >> a_I c$ (OFTEN VALID WHEN INCONDENSIBLE IS PRESENT IN VAPOUR):
  $T = T_{eq}$; $\dot{m} = a_I c(\tau - T_{eq})/\lambda$; $\dot{q}_I = (T_{eq} - \tau) a_I c$.

MPF 7	$\frac{8}{15}$	DISCUSSION OF THE EQUILIBRIUM INTERFACE TEMPERATURE, T_{eq}

- T_{eq} ($\equiv \tau_{eq}$) IS, BY DEFINITION, THE INTERFACE TEMPERATURE PREVAILING WHEN THE KINETIC RESISTANCE TO PHASE CHANGE IS ZERO.
- T_{eq} DEPENDS UPON THE PARTIAL PRESSURE OF THE PHASE-CHANGING MATERIAL IN THE VAPOUR PHASE, IN MANY CIRCUMSTANCES.
- IF THE VAPOUR PHASE IS PURE STEAM (SAY), T_{eq} DEPENDS ON THE PRESSURE p (WHICH IS ALSO THE PARTIAL PRESSURE).
- WHEN AN INCONDENSIBLE IS PRESENT (EG AIR IN STEAM), AN ADDITIONAL DIFFERENTIAL EQUATION (EG FOR THE AIR CONTENT) IS NEEDED IN GENERAL; BUT, IN SOME CIRCUMSTANCES (EG COOLING-TOWER FLOWS), IT CAN BE AVOIDED (SEE, EG SPALDING, 1963).

MPF 7	$\frac{9}{15}$	FINITE- DOMAIN FORM OF THE ENERGY EQUATIONS, 1

- FOR THE LIGHT PHASE, WITH THE NOTATION OF LECTURE 2:

$$\{h_P(\tfrac{r}{v})_P - h_{P-}(\tfrac{r}{v})_{P-}\}\frac{\delta x}{\delta t} + h_P\{[-\tfrac{r}{v}u_w] + [\tfrac{r}{v}u_e]\}$$
$$-h_w[\tfrac{r}{v}u_w] - h_E[-\tfrac{r}{v}u_e] = \{a_o(h_P - h_o) + a_I(h_P - h_I) + [\dot{m}]\, h_I\}\delta x$$

- NOTE THAT THE MASS-CONSERVATION PRINCIPLE IMPLIES:

$$\{(\tfrac{r}{v})_P - (\tfrac{r}{v})_{P-}\}\frac{\delta x}{\delta t} + \{[-\tfrac{r}{v}u_w] + [\tfrac{r}{v}u_e]\}$$
$$-[\tfrac{r}{v}u_w] - [-\tfrac{r}{v}u_e] = [\dot{m}]\ \delta x\ ;$$

SO THE MULTIPLIER OF h_P CAN BE EXPRESSED IN TERMS OF THE MULTIPLIERS OF h_w, h_E, h_o AND h_I.

MPF 7	$\frac{10}{15}$	FINITE- DOMAIN THE ENERGY EQUATIONS, 2

- DEFINITION: $\alpha_{P-} \equiv (r/v)_{P-}\,\delta x/\delta t$; $\alpha_W \equiv [(r/v)u_w]$;
  $\alpha_E \equiv [-(r/v)u_e]$; $\alpha_o \equiv a_o.\delta x$;
  $\alpha_I \equiv a_I.\delta x$; $\alpha_m \equiv [\dot{m}].\delta x$.
- CONSEQUENCE:

$$h_P = \frac{\alpha_{P-}h_{P-} + \alpha_W h_W + \alpha_E h_E + \alpha_o h_o + (\alpha_I + \alpha_m)\, h_I}{\alpha_{P-} + \alpha_W + \alpha_E + \alpha_o + \alpha_I + \alpha_m}$$

- HEAVY PHASE: A CORRESPONDING EQUATION CAN BE WRITTEN. OF COURSE IT IS $[-\dot{m}]H_I$ WHICH APPEARS THERE.
- COMMENT: α_m IS NOT KNOWN, IN GENERAL, UNTIL h_P HAS BEEN COMPUTED;
  BUT $\alpha_m << \alpha_I$, SO A GUESS WILL SUFFICE.

MPF 7	11/15	FINITE-DOMAIN FORMS OF THE ENERGY EQUATION, 3

- DETERMINATION OF h_I:

  h_I CORRESPONDS TO THE TEMPERATURE τ_I, WHICH IS A WEIGHTED AVERAGE OF τ_{eq}, τ_P AND T_P (SLIDE 6); USUALLY τ_I CAN BE TAKEN AS τ_{eq} (FOR β VERY LARGE; BUT THE GENERAL CASE CAN BE HANDLED).
- DETERMINATION OF $\dot{m}$:

  $\dot{m}$ CAN BE DEDUCED, ONCE τ_P AND T_P HAVE BEEN OBTAINED VIA h_P AND H_P, FROM THE EQUATION OF SLIDE 6 OR (FOR LARGE β) 7.
- IT IS THIS EQUATION THAT SHOWS THAT α_m/α_I IS OF THE ORDER OF $c(\tau - \tau_I)/\lambda_I$, IE<<1.
- USUALLY IT IS ONE OF THE EXTREMES, $\beta = \infty$ OR $\beta = 0$, THAT IS NEEDED IN PRACTICE.

MPF 7	12/15	SOLUTION PROCEDURE, 1: RECOMMENDED ORDER

(1) SOLVE FOR h_P AND H_P FROM ABOVE EQUATIONS.

(2) DEDUCE THE VALUES OF $\dot{m}$, $\dot{v}$ AND $\dot{V}$.

(3) SOLVE FOR R, FROM DENSE-PHASE CONTINUITY.

(4) OBTAIN GUESS OF PRESSURE, EG FROM JOINT MOMENTUM EQUATION.

(5) SOLVE FOR u. AND U. FROM INDIVIDUAL MOMENTUM EQUATIONS.

(6) SOLVE FOR p' FROM WEIGHTED JOINT CONTINUITY EQUATION.

(7) APPLY CORRESPONDING CHANGES TO u, U AND THE DENSITIES.

(8) RETURN TO (1) AND REPEAT UNTIL THE RESIDUALS IN THE EQUATIONS BECOME ACCEPTABLY SMALL.

MPF 7	13/15	SOLUTION PROCEDURE, 2: WHEN β EQUALS ZERO (NO MASS TRANSFER)

- IT IS CONVENIENT NOW TO WORK IN TERMS OF TEMPERATURES, τ AND T, RATHER THAN ENTHALPIES, h AND H.
- THE ENERGY EQUATION FOR THE LIGHT PHASE TAKES THE FORM:

$$\tau_P = \frac{\tau_{P-}\,\alpha_{P-} + \tau_W\,\alpha_W + \tau_E\,\alpha_E + \tau_O\,\alpha_O + T_P\,\alpha_T}{\alpha_{P-} + \alpha_W + \alpha_E + \alpha_O + \alpha_T}$$

- THE ENERGY EQUATION FOR THE DENSE PHASE TAKES A SIMILAR FORM, IE τ_P APPEARS IN IT ON THE RIGHT.
- IF α_T IS LARGE, "HOBBLING" OCCURS; IT IS NECESSARY TO EMPLOY THE PARTIAL-ELIMINATION ALGORITHM (PEA) TO ACCELERATE CONVERGENCE.

MPF 7	14/15	SOLUTION PROCEDURE, 3: THE TWO-VARIABLE ALGORITHM

- THE PROBLEM IS THAT OF SOLVING THE TWO SETS OF SIMULTANEOUS EQUATIONS: $dx_I = ax_{I+1} + bx_{I-1} + c + eX_I$

$$DX_I = AX_{I+1} + BX_{I-1} + C + Ex_I$$

- THE "PEA" PROCEEDS BY REPLACING THESE EQUATIONS BY:

$$(d - eE/D)x_I = ax_{I+1} + bx_{I-1} + c + (AX_{I+1} + BX_{I-1} + C)\ e/D$$

$$(D - Ee/d)X_I = AX_{I+1} + BX_{I-1} + C + (ax_{I+1} + bx_{I-1} + c)\ E/d$$

- "HOBBLING" IS NOW REDUCED, BUT STILL PRESENT TO SOME EXTENT.
- IT IS THEREFORE SOMETIMES WORTH WHILE TO EMPLOY THE TWO-VARIABLE ALGORITHM (TVA) WHICH SOLVES BOTH SETS OF EQUATIONS SIMULTANEOUSLY.
- THE TVA IS SIMPLY A STANDARD GAUSSIAN ELIMINATION PROCEDURE.

MPF 7	15/15	CONCLUDING REMARKS

- ALTHOUGH THERE ARE SOME CASES IN WHICH IT IS NOT NECESSARY (SEE LECTURE 6), IT IS COMMONLY REQUIRED TO SOLVE TWO ENERGY EQUATIONS IN ADDITION TO THE MOMENTUM AND CONTINUITY EQUATIONS.
- METHODS ARE AVAILABLE FOR DOING THIS WITHOUT DIFFICULTY.
- WHEN MASS TRANSFER AND HEAT TRANSFER ARE HINDERED BY THE PRESENCE OF A MIXTURE COMPONENT WHICH DOES NOT CHANGE PHASE, AN ADDITIONAL EQUATION MUST BE SOLVED FOR ITS CONCENTRATION.
- THE "PEA", "TVA" OR OTHER ADVANCED ALGORITHM MAY BE NEEDED WHEN THE EQUATIONS ARE TIGHTLY COUPLED.

MPF 8	1/15	LECTURE 8: GENMIX 2P: DETAILS OF ITS WORKINGS

CONTENTS

- SOLUTION FOR U(I)
- SOLUTION FOR H(I)
- COMPUTATION OF CROSS-STREAM DISTANCE, Y(I) ETC
- COMPUTATION OF TEE(I) AND AL(I)
- OBTAINING P(I) VIA "SNIP"
- SOLUTION FOR AL2(I)
- SOLUTION FOR U2(I)
- SOLUTION FOR H2(I)
- SOLUTION FOR R2(I)
- ADJUSTMENT OF R2(I)

NOTE: THE LISTING SHOULD BE INSPECTED DURING THIS LECTURE.

MPF 8	2/15	SOLUTION FOR U(I), AS IN STANDARD GENMIX (SPALDING, 1977).

- SOLVE, an ENTRY in COMP, is called from MAIN, Chapter 8.
- Convection terms (CON, HCON) are computed if neither RMI nor RME is very small.
- PHYSU, an ENTRY in PHYS, is called from COMP to supply effective viscosities EMU(I) and source terms SI(I), SIP(I).
- Diffusion terms (DIFU) are computed, differently according to the value of KRAD.
- Coefficients A and B are computed, differently according to presence or absence of convection.

MPF 8	$\frac{3}{15}$	SOLUTION FOR U(I) CONCLUDED

- Coefficients C and D are computed (Note difference from GENMIX 0: interphase friction requires finite SIP).
- For free boundaries (KIN = KEX = 2), U(1) and U(N) are computed from pressure gradient.
- Downstream values of U(I) are computed via TDMA (two DO loops; note that C and D arrays are used to store intermediate coefficients in first loop).
- Wall shear stresses are computed for KIN = 1, KEX = 1.
- RETURN to MAIN if NF.LT.1 (it is not).

NOTE:
Apart from finite SIP, procedure is as in GENMIX 0.

MPF 8	$\frac{4}{15}$	SOLUTION FOR H(I) IE FOR F(I,JH) ≡ F(I,1) AS IN STANDARD GENMIX.

- F-Section of COMP follows U section.
- PHYSF, an ENTRY in PHYS, is called after index manipulation.
- WALL is called in case of need (KIN = 1 or KEX = 1).
- New A's and B's are used, unless those of U will do (σ = 1).
- C's and D's are computed. KSOURC = 1 in present case.
- Downstream values are computed via the TDMA which solves: $D\phi_I = A\phi_{I+1} + B\phi_{I-1} + C_I$.
- Boundary-value adjustments are made if walls are present.
- RETURN to MAIN if NF.EQ.1 (true in case illustrated).

MPF 8	$\frac{5}{15}$	COMPUTATION OF CROSS-STREAM DISTANCE, Y(N)

- In Chapter 9 of MAIN, $r_1\rho_1$ is placed in RHO(I), because the single-phase COMP is being used.
- DISTAN, an ENTRY in COMP, is called.
- $1/(r_1\rho_1u_1)$ is computed in COMP, according to value of NOVEL.
- ADPEI(I) is computed as $\delta\omega/(r_1\rho_1u_1)$.
- Y(I) and R(I) are computed from ADPEI(I) variously according to KRAD, CSALFA, R(1) etc.
- RETURN to MAIN.
- ρ_1 is replaced in RHO(I).
- Note: For KRAD = 1, R(I) is set equal to 1.0.

MPF 8	$\frac{6}{15}$	COMPUTATION OF YINT(I)

- FOR PLANE FLOW (KRAD = 1),

$$y_{I+\frac{1}{2}} = (y_I + \xi\, y_{I+1})/(1+\xi)$$

WHERE $\xi \equiv (r_1\rho_1u_1)_I/(r_1\rho_1u_1)_{I+1}$

- FOR AXISYMMETRICAL FLOW:

$$y_{1/2} = |(y_I^{\,2} + \xi\, y_{I+1}^2)/(1+\xi)|^{1/2}$$

- YINT(I) ≡ $y_{I+\frac{1}{2}}$
- THE RELEVANT CODING SEQUENCE IS IN SUBROUTINE ADJ, CALLED FROM MAIN, AFTER THE RETURN FROM DISTAN & RESTORATION OF RHO's.
- RECRU STILL CONTAINS $1./(r_1\rho_1u_1)$.

MPF 8	7/15	COMPUTATION OF GRID-LINE ANGLES, TEE(I)

- TEE(I) = TANI + (YINT(I) - YINTU(I))/DX in the DO 912 loop in Subroutine ADJ.
- TANI is the tangent of the inclination of the I boundary to the direction of the x-axis at x = 0.
- YINT (and Y) are measured from this boundary.
- YINTU has been stored in Chapter 6 of MAIN in the DO 61 loop.
- TEE(I) is the inclination of the constant-ω line, not of the constant-ϕ_1 line.

MPF 8	8/15	COMPUTATION OF PHASE-1 STREAMLINE ANGLES, AL(I)

- AL(I) would equal TEE(I) if there were no cross-flow (RMI = RME = 0).
- AL(I) therefore equals TEE(I) plus an increment allowing for cross-flow, which takes account of local $r_1\rho_1 u_1$.
- r_1 is obtained, here and everywhere, as $1 - r_2$.
- The relevant coding sequence is in Subroutine ADJ, at the DO 920 loop.
- AL(I) is here the tangent of the streamline angle, but it is not distinguished from the angle or its sine.

MPF 8	9/15	OBTAINING P(I) VIA "SNIP"

- Since AL(I) has been obtained from continuity, the lateral pressure distribution P(I) can be obtained from it.
- This is performed by the DO 930 loop of Subroutine ADJ.
- Terms accounted for are all those in the lateral (y-direction) momentum equation:
  convection of momentum from upstream (hence ALU);
  lateral convection (hence FLUXS, FLUXN);
  interphase friction (hence HFIP, AL2).
- P(1) and P(2) are set arbitrarily to zero, and P(N) is set equal to P(NM1).

MPF 8	10/15	SOLUTION FOR AL2(I)

- COMP2P is called at the end of Chapter 10 of MAIN
- AL2(I) is computed in Chapter 1 of COMP2P.
- Coefficients A, B, C and D are computed in DO 100.
- A and B represent the effects of transport of phase 2 across constant-ω lines
  (hence AL2(I) + AL2(I+1) - TEE(I) - TEE(I+1).
- C represents the effects of convection from upstream, of pressure gradient, of gravity, and of interphase friction.
- D represents the effect of outflow from the cell.
- Downstream AL2's are produced by the TDMA loops DO 101 and DO 102.

MPF 8	11/15	SOLUTION FOR U2(I)

- U2's are computed in Chapter 2 of COMP2P.
- Coefficients A, B, C and D are assembled in DO 200.
- A and B represent the effect of phase-2 transport across constant-ω lines (hence AL2(I) - TEE(I)).
- C represents the effects of convection from upstream, of pressure gradient, of gravity, and of interphase friction.
- D represents the effect of outflow from the cell.
- Some information for later use is saved in ASTORE(I) and CSTORE(I).
- Downstream U2's are computed by TDMA loops DO 201 and DO 202, unless TRNSNT is .TRUE..

MPF 8	12/15	SOLUTION FOR H2(I)

- H2's are computed in Chapter 3 of COMP2P.
- A's and B's are identical with those for U2's and therefore need not be recalculated.
- C(I) uses the contents of CSTORE(I) to compute the enthalpy transported from upstream (hence .H2U(I)).
- C(I) also contains a contribution from interphase heat transfer (hence H(I)).
- D(I) represents the effects of outflow from the cell.
- Downstream H2's are computed by the TDMA in DO 301 and DO 302.

MPF 8	13/15	SOLUTION FOR R2(I)

- R2's are computed in Chapter 4 of COMP2P.
- A's and B's are similar to those for U2 and H2, but without the R2 (hence ASTORE is used).
- C's represent the effects of upstream convection, and are equal to CSTORE's.
- D's represent the effect of outflow from the cell; they cannot be computed from the sum of A, B and inflow terms in this case.
- Downstream R2's are produced by the TDMA loops DO 401 and DO 402; but a point-by-point procedure appears as a preferred alternative at DO 411 to which control passes if PBP = .TRUE..

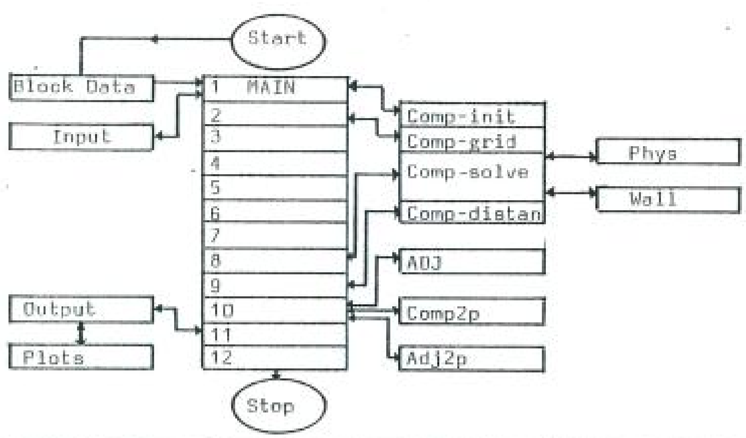
MPF 8	14/15	ADJUSTMENT OF R2(I)

- ADJ2P is called from Chapter 10 of MAIN.
- In Chapter 1 of ADJ2P, R2's are compared with R2LAST's, stored in COMP2P before new R2's were obtained.
- R2 is also forced to stay within the limits 0.0 and 0.99.
- The maximum error is put in ERROR(ITER).
- RETURN takes place at once for METHOD = 1.
- For METHOD = 2, R2's are under-relaxed in Chapter 2.
- For METHOD = 3, pressure corrections are computed in Chapter 3 via formulation of A's, B's, C's and D's and TDMA solution; then new R2's are computed.

MPF 8 | 15/15 | CONCLUDING REMARKS

- GENMIX 2P IS A NEW CODE, CAPABLE OF MANY IMPROVEMENTS, ESPECIALLY FROM THE POINT OF VIEW OF ECONOMY.
- AN OPEN (AND REPETITIOUS) CODING STYLE HAS BEEN ADOPTED IN THE NEW SUBROUTINES, FOR THE SAKE OF CLARITY.
- IF INTERPHASE MASS TRANSFER TAKES PLACE, ACCOUNT MUST BE TAKEN OF THE FACT THAT GRID-POINT VALUES OF ω WILL VARY WITH x, NOT REMAIN CONSTANT AS AT PRESENT.

MPF 8 | APPENDIX | FLOW DIAGRAM OF GENMIX 2P



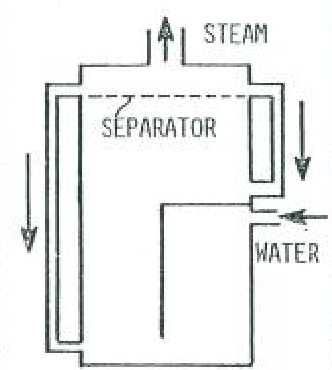
MPF 9 | 1/15 | LECTURE 9: MULTI-DIMENSIONAL TWO-PHASE FLOWS

CONTENTS

- THE STEAM-GENERATOR PROBLEM
  - DIFFERENTIAL AND FINITE-DIFFERENCE EQUATIONS
  - AUXILIARY RELATIONS
  - SOLUTION PROCEDURE
- PROBLEMS WITH SPATIALLY SEPARATED PHASES
  - THE CONTINUITY EQUATION
  - THE GALA METHOD OF SOLUTION
  - EXAMPLES
- CONCLUDING REMARKS

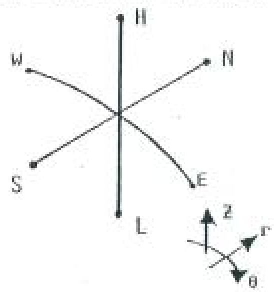
MPF 9 | 2/15 | THE STEAM-GENERATOR PROBLEM: DESCRIPTION

- THE STEAM IS FORMED ON THE SHELL SIDE OF THE EXCHANGER.
- THE SHELL CONTAINS MANY TUBES, FILLED WITH HOT WATER FROM A NUCLEAR REACTOR. THESE ARE NOT SHOWN IN THE SKETCH.
- DOWNCOMERS ALLOW WATER, SEPARATED FROM STEAM, TO RE-ENTER THE BOTTOM OF THE SHELL.
- THE FLOW IS 3D, 2-PHASE WITH SLIP, RESISTED, TRANSIENT, COMPRESSIBLE.



MPF 9	3/15	THE STEAM-GENERATOR PROBLEM: THREE-DIMENSIONALITY

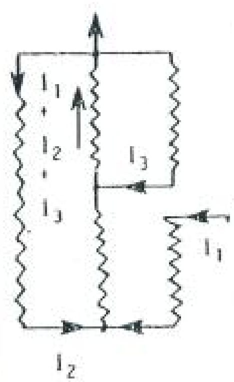
- THERE ARE NOW 3 VELOCITY COMPONENTS TO COMPUTE FOR EACH PHASE: u_z, u_r, u_θ, U_z, U_r, U_θ.
- EACH GRID POINT IS CONNECTED WITH 6 NEIGHBOURS INSTEAD OF 2.



- FOR CONVENIENCE OF ORGANISATION OF THE SUCCESSIVE-ADJUSTMENT PROCEDURE, ATTENTION PASSES UPWARD FROM ONE CONSTANT-z SLAB TO THE NEXT, REPEATEDLY.
- PLANE-WISE "BLOCK ADJUSTMENTS" MAY BE ADVANTAGEOUS IN ORDER TO ACCELERATE CONVERGENCE.

MPF 9	4/15	THE STEAM-GENERATOR PROBLEM: TREATMENT OF THE DOWNCOMERS

- THE GENERATOR OF PANEL 2 CAN BE REGARDED AS CONSISTING OF THE CIRCUIT SHOWN.
- CONVERGENCE OF THE SOLUTION PROCEDURE CAN BE GREATLY ACCELERATED BY MAKING FREQUENT CIRCULATION ADJUSTMENTS.
- THESE INVOLVE INTEGRATING THE SECTION-AVERAGE MOMENTUM EQUATIONS AROUND EACH LOOP, NOTING THE PRESSURE IMBALANCE, AND ADJUSTING THE LOOP FLOW UNTIL IT DISAPPEARS.
- THIS TECHNIQUE IS A DEVELOPMENT OF SNIP.



MPF 9	5/15	THE STEAM-GENERATOR PROBLEM: TREATMENT OF THE PRESENCE OF TUBES

- THE "PRIMARY-FLUID" TUBES REDUCE THE VOLUMES AND AREAS AVAILABLE FOR OCCUPANCY AND FLOW, SO THAT, FOR EXAMPLE: $1 - r - R = \beta$ WHERE $\beta \equiv$ BLOCKAGE FACTOR.
- AREAS MAY BE REGARDED AS REDUCED IN RATIO $1 - \beta$; BUT, STRICTLY SPEAKING, THE AREA BLOCKAGE RATIO DIFFERS FROM β, AND IS DIFFERENT FOR EACH DIRECTION.
- THE TUBES EXERT RESISTANCES TO FLOW, APPEARING AS MOMENTUM-SOURCE TERMS (NEGATIVE), DEPENDENT ON VELOCITY, ETC.
- THESE RESISTANCES ARE EXPRESSED IN LINEARIZED FORM AND APPEAR IN:

$$u_P = \frac{\ldots\ldots\ldots\ldots + K u_{P*}}{\ldots\ldots\ldots\ldots + K}$$

MPF 9	6/15	THE STEAM-GENERATOR PROBLEM: TREATMENT OF THE HEAT TRANSFER

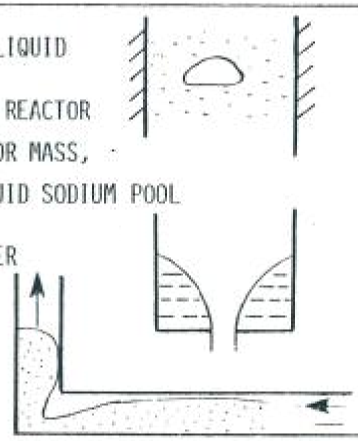
- THE PRIMARY-FLUID TEMPERATURES ARE BEST CALCULATED BY INTEGRATION ALONG THE TUBES, WITH ESTIMATED SHELL-SIDE TEMPERATURE, r, R, ETC. THE RESULT IS AN ESTIMATED $\dot{q}$ DISTRIBUTION.
- THE $\dot{q}$ DISTRIBUTION CAN THEN BE USED AS THE BASIS OF A CALCULATION OF THE SHELL-SIDE DISTRIBUTIONS OF h, r, R, u_z, u_r, u_θ, U_z, U_r, U_θ, p.
- THEN ITERATION ENSUES, AND LEADS TO RAPID CONVERGENCE, BECAUSE $\dot{q}$ IS NOT VERY SENSITIVE TO THE SMALL CHANGES IN SHELL-SIDE CONDITIONS WHICH ARE ENCOUNTERED.

MPF 9	7/15	THE STEAM-GENERATOR PROBLEM: PRESENT STATUS

- THE COMBINATION OF CIRCULATION ADJUSTMENT WITH POINT-BY-POINT SATISFACTION OF CELL-WISE CONSERVATION EQUATIONS HAS PROVED TO BE SATISFACTORY FROM THE POINT OF FIEW OF ECONOMY AND ACCURACY (NUMERICAL); BUT THE p'-EQUATIONS ARE BEST SOLVED IN A 3D WHOLE-FIELD MANNER.
- PREDICTIONS ARE QUALITATIVELY IN AGREEMENT WITH INFERENCES FROM OBSERVATIONS ON FULL-SIZE GENERATORS (e.g., AS TO WHERE EXCESSIVE VELOCITIES OCCUR).
- THE MAJOR NEED IS FOR RELIABLE INFORMATION ON WHAT INTERPHASE-FRICTION FORMULAE ARE APPROPRIATE UNDER VARIOUS CIRCUMSTANCES. EXPERIMENTS ARE NEEDED.
- AT PRESENT, CALCULATION MUST PROCEED BY WAY OF UPPER AND LOWER LIMITS.

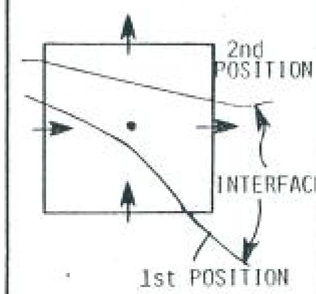
MPF 9	8/15	FULLY-SEPARATED FLUIDS: SOME PRACTICAL EXAMPLES

- THE RISE OF A BUBBLE IN A LIQUID
- THE EXPLOSION OF A NUCLEAR REACTOR CORE, FORMING A SODIUM VAPOR MASS, AT HIGH PRESSURE, IN A LIQUID SODIUM POOL
- THE DRAINAGE OF A TANK UNDER ZERO-GRAVITY CONDITIONS.
- THE FORMATION OF LIQUID SLUGS IN PIPELINES



MPF 9	9/15	FULLY-SEPARATED FLOW: THE NUMERICAL-ANALYSIS DIFFICULTY

- THERE IS AN INTERFACE ACROSS WHICH THE DENSITY CHANGES SHARPLY.
- THE FINITE-DOMAIN FORM OF THE MASS-CONTINUITY EQUATION INVOLVES $\iint \rho u_x dy\, dt$ and $\iiint \frac{\partial \rho}{\partial t} dt\, dy\, dx$; THESE ARE DIFFICULT TO EVALUATE SO AS TO ENSURE:
  - CELL-TO-CELL COMPATIBILITY;
  - OVERALL CONSERVATION;
  - ERROR-FREE PROGRAMMING.



MPF 9	10/15	FULLY-SEPARATED-FLOW: ALTERNATIVE CONTINUITY EQUATIONS

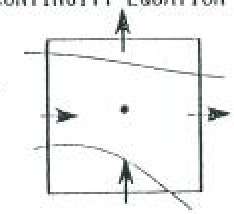
- THE FOLLOWING ARE EQUIVALENT:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \vec{u}) = 0; \text{ AND } \mathrm{div}\, \vec{u} = - \frac{D\ln\rho}{Dt}$$

- $-D\ln\rho/Dt$ REPRESENTS THE RATE OF CHANGE OF SPECIFIC VOLUME ($\equiv 1/\rho$) OF THE MATERIAL AS IT PASSES THROUGH SPACE: IT IS EASY TO CALCULATE, AND DEPENDS ON p AND T, NOT $\vec{u}$.
- FOR EXAMPLE, IF THE TWO FLUIDS HAVE ρ's WHICH, THOUGH UNEQUAL, ARE EACH CONSTANT, $D\ln\rho/Dt$ EQUALS ZERO; THEN $\mathrm{div}(\vec{u}) = 0$.

MPF 9	11/15	FULLY-SEPARATED FLOW: THE GALA TECHNIQUE

- GALA STANDS FOR GAS AND LIQUID ANALYZER.
- THE SECOND (VOLUMETRIC) FORM OF THE CONTINUITY EQUATION IS USED.
- THEREFORE THERE IS NO NEED TO CALCULATE $\iint \rho u \, dy \, dt$, ONLY $\iint u \, dy \, dt$, WHICH IS MUCH EASIER.
- THE EXPRESSION $\iiint -\frac{D \ln \rho}{Dt} \, dt \, dx \, dy$ IS ALSO EASY TO COMPUTE AS A RULE.
- USUALLY IT DEPENDS ONLY ON PRESSURE CHANGE AND ON HEAT INPUT.

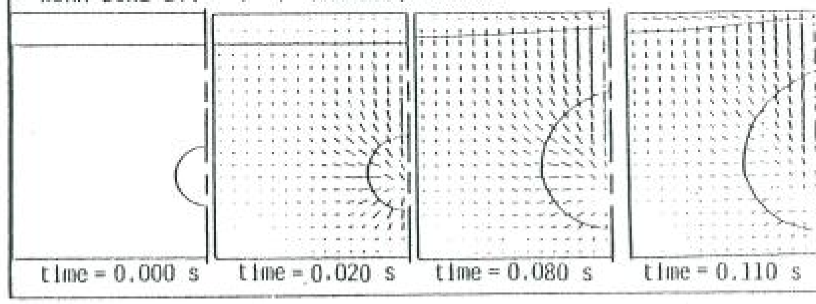


MPF 9	12/15	FULLY SEPARATED FLOWS: FURTHER REMARKS ABOUT GALA

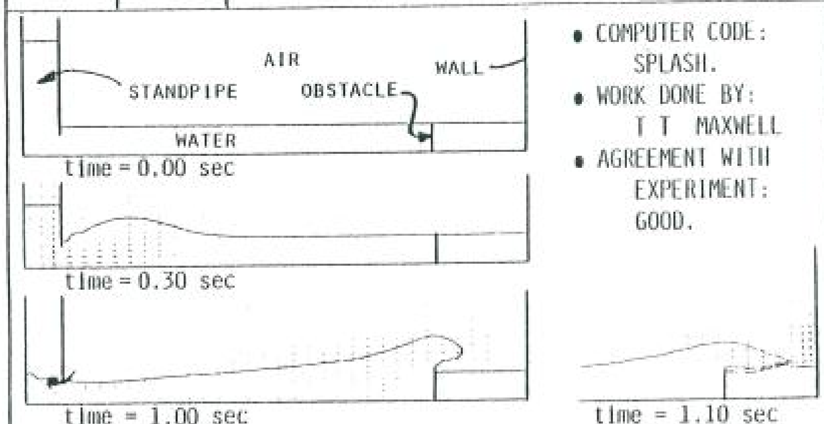
- IN THE MOMENTUM EQUATIONS, THE FORMATION OF INTEGRALS SUCH AS $\iint \rho u^2 \, dy \, dt$ IS STILL REQUIRED. HOWEVER, APPROXIMATIONS BRING NO SERIOUS INCOMPATIBILITIES HERE.
- THE POSITION OF THE INTERFACE IS BEST DETERMINED BY "PARTICLE TRACKING": PARTICLES ARE PLACED ALONG THE INTERFACE AT THE START; THEN THEIR LOCATIONS ARE UP-DATED AT THE END OF EACH TIME INTERVAL IN ACCORDANCE WITH: $\frac{D\vec{x}}{Dt} = \vec{u}_p$
- INTERPOLATION BETWEEN VELOCITY NODES, TO YIELD $\vec{u}_p$, MUST BE CAREFULLY PERFORMED.
- CARE IS NEEDED IN THE USE OF ALL THE PARTICLE-LOCATION INFORMATION FOR THE PROCUREMENT OF THE VOLUME FRACTIONS OF THE PHASES IN THE INTERSECTED CELLS.

MPF 9	13/15	FULLY SEPARATED FLOWS: THE EXPANDING-BUBBLE PROBLEM

- STARTING CONDITION: BUBBLE AT HIGH PRESSURE.
- FLUIDS: INCOMPRESSIBLE LIQUID; COMPRESSIBLE GAS.
- COMPUTER CODE: SPLASH (SPECIAL PROGRAM FOR LIQUID-AIR SYSTEM HYDRODYNAMICS).
- WORK DONE BY: T T MAXWELL, (1977).



MPF 9	14/15	FULLY SEPARATED FLOW THE BREAKING WAVE



- COMPUTER CODE: SPLASH.
- WORK DONE BY: T T MAXWELL
- AGREEMENT WITH EXPERIMENT: GOOD.

MPF 9	15/15	CONCLUDING REMARKS

- THE TECHNIQUES DESCRIBED IN EARLIER LECTURES WORK WELL FOR TWO- AND THREE-DIMENSIONAL PHENOMENA ALSO.
- SLAB-WISE AND CIRCUIT-ADJUSTMENT TECHNIQUES ARE NEEDED IN ORDER TO PROMOTE ECONOMY.
- THE "GALA" TECHNIQUE GREATLY SIMPLIFIES THE TREATMENT OF THE DENSITY DISCONTINUITIES ARISING IN SEPARATED-PHASE PROBLEMS.
- THE TECHNIQUES HAVE ONLY JUST BEGUN TO BE APPLIED TO PRACTICAL PROBLEMS; RAPID PROGRESS IS NOW TO BE EXPECTED.

MPF 10	1/15	LECTURE 10: TWO-PHASE FLOW WITH CHEMICAL REACTION

CONTENTS

- 2D STEADY PARABOLIC PROCESSES
- 1D UNSTEADY PROCESSES
- 2D STEADY ELLIPTIC PROCESSES
- 3D STEADY ELLIPTIC PROCESSES
- 2D AND 3D UNSTEADY PROCESSES
- THE NEED FOR EMPIRICAL INPUT DATA
- THE NEED FOR EXPERIMENTAL VALIDATION
- THE NEED FOR MULTI-PHASE MODELS

NOTES: THE PURPOSE OF THE LECTURE IS TO PROMOTE THE APPLICATION OF MULTI-PHASE-FLOW MODELLING TO COMBUSTION PROCESSES.

MPF 10	2/15	2D STEADY PARABOLIC PROCESSES

(1) THE IGNITION AND BURNING OF AN AXI-SYMMETRICAL COAL-AIR JET INJECTED INTO A FURNACE.

- VARIABLES: VELOCITY COMPONENTS, TEMPERATURE, OXYGEN CONCENTRATION OF AIR, VELOCITY COMPONENTS, TEMPERATURE AND WEIGHT OF PARTICLES OF COAL (TREATED AS UNIQUE SUBSTANCE); RADIATION-FLUX SUM (TWO-FLUX MODEL); TURBULENCE PARAMETERS.
- CODE: GENMIX 2P (WITH ALLOWANCE FOR MASS TRANSFER).
- UTILITY: PROMOTION OF UNDERSTANDING OF THE PROCESS, FOLLOWED BY USE IN IMPROVEMENTS OF DESIGN AND OPERATION.
- PROBLEMS: A MORE COMPLEX MASS-TRANSFER-KINETICS EXPRESSION IS NEEDED THAN WAS INDICATED IN LECTURE 7.

MPF 10	3/15	2D STEADY PARABOLIC PROCESSES

(2) THE BURNING OF AN AXI-SYMMETRICAL JET OF LIQUID-FUEL DROPLETS FROM AN AIR-ATOMISING INJECTOR.

- VARIABLES: VELOCITY COMPONENTS, TEMPERATURE, FUEL-VAPOUR AND OXYGEN CONCENTRATIONS OF THE GAS; VELOCITY COMPONENTS, TEMPERATURE AND DIAMETER OF DROPLETS; TURBULENCE PARAMETERS.
- CODE: GENMIX 2P (WITH ALLOWANCE FOR MASS TRANSFER).
- PROBABLE RESULTS: THE INFLUENCE OF DROPLET SIZE ON FLAME LENGTH WILL BE PREDICTED (CORRECTLY?); ALSO THE INFLUENCES OF PRE-HEATING OF LIQUID, OF FUEL PROPERTIES, AND OF CHEMICAL KINETICS.
- PRACTICAL DIFFICULTY: KNOWING WHAT DROPLET SIZE TO PRESUME AT THE START.

MPF 10	4/15	1D UNSTEADY PROCESSES

(3) 1D OSCILLATIONS IN A LIQUID-PROPELLANT ROCKET MOTOR, AS INDICATED IN LECTURE 6.

(4) "BUZZ" IN THE REHEAT PIPE OF AN AIR-BREATHING JET ENGINE, AS INDICATED IN LECTURE 6, WITH THE BURNING AND UNBURNED GAS STREAMS BEING TREATED AS DISTINCT PHASES.

(5) START-UP AND SHUT-DOWN TRANSIENTS IN A FLUIDISED BED.

- VARIABLES: VELOCITY, TEMPERATURE, COMPOSITION OF GAS; VELOCITY, TEMPERATURE, COMPOSITION OF SOLID.
- UTILITY: THE INFLUENCES OF PARTICLE SIZE AND REACTIVITY ON TRANSIENT BEHAVIOUR COULD BE EXPLORED.
- PROBLEM: FLUIDIZED BEDS RARELY ARE ONE-DIMENSIONAL.

MPF 10	5/15	1D UNSTEADY PROCESSES

(6) EXPLOSION IN A DUSTY GAS CONTAINED IN A PIPE.

- VARIABLES: VELOCITY, TEMPERATURE AND COMPOSITION OF GAS; VELOCITY, TEMPERATURE AND WEIGHT OF PARTICLES: RADIATION FLUX.
- CODE: GENMIX 2P OR PLANT
- WHAT COULD BE EXPLORED:
  - INFLUENCES OF PARTICLE CONCENTRATION, PARTICLE SIZE, REACTIVITY OF SOLID, INITIAL OXYGEN CONCENTRATION, INITIAL TEMPERATURES, HEAT LOSSES TO WALLS, ON FLAME SPEED.
  - CONDITIONS FOR FLAMMABILITY.
  - CONDITIONS FOR TRANSITION TO DETONATION.
- PROBLEMS: ASSIGNING SUITABLE REACTION-KINETIC CONSTANTS TO PRACTICALLY OCCURRING DUSTS.

MPF 10	6/15	2D STEADY ELLIPTIC PROCESSES

(7) FLUIDISED-BED COMBUSTOR:

- VARIABLES: VELOCITY COMPONENTS, COMPOSITION, TEMPERATURE OF GAS; DITTO OF PARTICLES; TEMPERATURE DISTRIBUTION IN METAL WALLS & INTERNAL COOLANT FLUID OF TUBE-BUNDLE USED FOR HEAT EXTRACTION; RADIATION-FLUX SUMS.
- CODE: EG ONE SIMILAR TO THAT FOR THE STEAM GENERATOR.
- WHAT COULD BE EXPLORED:
  - INFLUENCE OF NON-UNIFORMITIES IN AIR INJECTION AT BASE OF REACTOR ON CIRCULATION WITHIN REACTOR AND ON CONSEQUENT HEAT-FLUX DISTRIBUTION.
  - ROLES OF PARTICLE SIZE, OF TUBE PITCH, DIAMETER & LOCATION, & OF HEIGHT/WIDTH RATIOS OF REACTOR, ON SENSITIVITY TO SUCH INFLUENCES.
- PROBLEMS: ASSIGNING THE CORRECT PARTICLE-TO-WALL TRANSFER LAWS.

MPF 10	7/15	3D STEADY ELLIPTIC PROCESSES

(8) THE PULVERIZED-COAL-BURNING POWER-STATION FURNACE.

- GEOMETRY: TALL RECTANGULAR-SHAPED CAVITY; WATER-COOLED WALLS; EXIT TO SUPER-HEATER AT TOP; INJECTION OF COAL & AIR THROUGH ONE WALL AT SEVERAL POINTS.
- VARIABLES: VELOCITIES, TEMPERATURE, COMPOSITION OF GAS; DITTO OF PARTICLES; TURBULENCE PARAMETERS; RADIATION-FLUX SUMS.
- WHAT COULD BE EXPLORED:
  - INFLUENCES OF INPUT PARAMETERS ON BURN-OUT EFFICIENCY, PARTICLE FALL-OUT, HEAT-FLUX PATTERNS.
  - DIFFERENCES IN PERFORMANCE RESULTING FROM CHANGE OF FUEL PROPERTIES, PULVERIZING FINENESS, ETC.

MPF 10	8/15	3D STEADY ELLIPTIC PROCESSES

(9) THE LIQUID-FUEL-BURNING GAS TURBINE.

- GEOMETRY: ANNULAR CAVITY; WALLS PERFORATED BY AIR-ENTRY HOLES; AIR AND DROPLET INJECTION AT ONE END; EXIT TO TURBINE AT OTHER END.
- VARIABLES: VELOCITIES, TEMPERATURE, COMPOSITION OF GAS; VELOCITIES, TEMPERATURE, SIZE, OF DROPLETS; TURBULENCE PARAMETERS.
- WHAT COULD BE EXPLORED:
  - INFLUENCE OF INJECTOR DESIGN ON IMPINGEMENT OF DROPLETS ON WALLS, OR EXIT OF UNBURNED LIQUID TO TURBINE.
  - ROLES OF FUEL AND ATOMISATION PROPERTIES IN DETERMINING UNIFORMITY OF OUTLET TEMPERATURE.

MPF 10	9/15	2D AND 3D UNSTEADY PROCESSES

(10) TRANSIENT PERFORMANCE OF FLUIDIZED-BED REACTORS.

- VARIABLES: AS FOR EXAMPLE (7) (SLIDE 6).
- WHAT COULD BE EXPLORED:
  - OPERATING TRANSIENTS (START-UP, SHUT-DOWN, CHANGE OF THROUGHPUT, CHANGE OF COOLANT TEMPERATURE).
  - INSTABILITIES, SUCH AS LARGE-BUBBLE FORMATION.
- REMARKS:
  - FLUIDIZED-BEDS ALWAYS EXHIBIT CONTINUOUS FLUCTUATION IN THEIR FLOW PATTERNS.
  - LITTLE UNDERSTANDING OF WHAT CAUSES THEM, OR OF WHAT COULD DIMINISH THEM, EXISTS AT PRESENT.
  - NUMERICAL MODELLING COULD HELP.

MPF 10	10/15	2D AND 3D UNSTEADY PROCESSES

(11) LIQUID-PROPELLANT ROCKET MOTOR INSTABILITY.

- GEOMETRY: THE ROCKET MOTOR CONSISTS OF AN APPROXIMATELY CYLINDRICAL CAVITY WITH AN INJECTOR PLATE AT ONE END AND THE CONVERGENCE TO THE NOZZLE AT THE OTHER. THE WALL IS COOLED.
- VARIABLES: VELOCITY COMPONENTS AND TEMPERATURES OF GAS AND LIQUID; COMPOSITION OF FORMER; DROPLET SIZE OF LATTER.
- WHAT COULD BE EXPLORED:
  - WHETHER IMPOSED OSCILLATIONS IN FLOWS DIE OUT IN THE COURSE OF TIME, OR INCREASE IN AMPLITUDE.
  - THE DEPENDENCE OF ATTENUATION/AMPLIFICATION LIMIT ON FUEL PROPERTIES, INJECTOR DESIGN, ETC.
  - HOW TO DESIGN HIGHLY STABLE MOTORS.

MPF 10	11/15	2D AND 3D UNSTEADY PROCESSES

(12)COMBUSTION IN THE DIESEL ENGINE.

- GEOMETRY: A LIQUID-FUEL SPRAY IS INJECTED INTO MOVING AIR IN A SPACE OF COMPLEX AND TIME-VARYING SHAPE.
- VARIABLES: AS FOR ROCKET MOTOR.
- WHAT COULD BE EXPLORED:
  - INFLUENCES OF ATOMISATION CHARACTERISTICS & OF VELOCITY & TIMING OF INJECTION ON ALL PERFORMANCE ASPECTS: IGNITION, BURN-OUT, ETC.
  - ROLE OF FUEL PROPERTIES.
- COMMENTS:
  - DIESEL-ENGINE COMBUSTION REPRESENTS MOST COMPLEX OF ALL COMBUSTION PROCESSES DISCUSSED SO FAR.
  - EVEN THOUGH COMPLETE MODELLING IS STILL OUT OF REACH, USEFUL INVESTIGATIONS OF PARTIAL MODELS ARE BEING MADE.

MPF 10	12/15	THE NEED FOR EMPIRICAL INPUT DATA

- INITIAL AND BOUNDARY CONDITIONS:
  THE INITIAL PARTICLE SIZES, TEMPERATURES & VELOCITIES MUST BE KNOWN.
- CHEMICAL-KINETIC INFORMATION:
  DATA ARE NEEDED FOR DEFINING THE RATES OF REACTION WITHIN THE GAS PHASE, AT THE PARTICLE-GAS INTERFACE (FOR SOLIDS) AND EVEN WITHIN THE PARTICLES.
- RADIATIVE-TRANSFER INFORMATION:
  EMMISSIVITIES ETC ARE NEEDED FOR GAS AND PARTICLES AS FUNCTIONS OF TEMPERATURE AND WAVE LENGTH.
- TURBULENCE INFORMATION:
  KNOWLEDGE IS REQUIRED OF HOW PARTICLES INFLUENCE, AND ARE INFLUENCED BY, THE SCALE AND ENERGY OF GAS TURBULENCE.

MPF 10	13/15	THE NEED FOR EXPERIMENTAL VALIDATION

- NUMEROUS EXPERIMENTS HAVE BEEN CONDUCTED AND REPORTED ON TWO-PHASE COMBUSTION PROCESSES OF THE KIND DISCUSSED.
- BECAUSE NUMERICAL MODELS, HAVE NOT BEEN AVAILABLE, THE DATA HAVE BEEN USEFUL ONLY TO ILLUMINATE THE PARTICULAR EXPERIMENTAL PROCESS UNDER STUDY.
- NOW THAT NUMERICAL MODELS, AND HYPOTHESES FOR THE INPUT TO THEM, ARE BECOMING AVAILABLE, IT IS DESIRABLE THAT THE REPORTED EXPERIMENTS SHOULD BE EXTENSIVELY PROCESSED, LEADING TO VALIDATION OR REFINEMENT OF THE MODEL.
- NO DOUBT, FRESH EXPERIMENTAL DATA, DESIGNED ESPECIALLY FOR MODEL VALIDATION, WILL PROVE NECESSARY THEREAFTER.

MPF 10	14/15	CONCEPTUAL DIFFICULTIES

- ALL PARTICLES AT ONE LOCATION DO NOT HAVE THE SAME SIZE OR TEMPERATURE.
- ALL PARTICLES OF THE SAME SIZE AND TEMPERATURE AT ONE LOCATION DO NOT HAVE THE SAME VELOCITY COMPONENTS.
- FOR FLOWS IN WHICH THE CONDENSED PHASE IS DILUTE, COMMON IN COMBUSTION PROCESSES, EQUILIBRIATION PROCESSES (EG MOMENTUM, HEAT OR MATTER EXCHANGE) ARE EXTREMELY SLOW.
- TO SPEAK OF THE DROPLET SIZE, TEMPERATURE x-WISE VELOCITY, VAPORIZATION RATE, ETC, THEREFORE IS A DUBIOUS PRACTICE.

MPF 10 | 15/15 | THE NEED FOR MULTI-PHASE MODELS OF COMBUSTION PROCESSES

- TO DISTINGUISH THERMODYNAMIC PHASE DIFFERENCES IS NOT ENOUGH: THE PARTICULATE PHASE NEEDS FURTHER SUB-DIVISION.
- MANY PRINCIPLES OF SUB-DIVISION ARE POSSIBLE, EG:
  - SPLIT BETWEEN UPWARD-MOVING AND DOWNWARD-MOVING;
  - 1D SPLIT INTO GROUPS OF VARIOUS PARTICLE-SIZE RANGES;
  - 2D SPLIT INTO GROUPS OF VARIOUS SIZE & TEMPERATURE RANGES;
  - 3D SPLIT INTO GROUPS OF VARIOUS U, V AND W RANGES;
  - 4D, 5D ETC SPLITS.
- EACH GROUP CAN BE REGARDED AS A DISTINCT PHASE.
- COMPUTER-STORAGE AND -TIME LIMITATIONS NECESSITATE THAT MULTIPLICATION OF PHASES SHOULD BE UNDERTAKEN SPARINGLY.

MPF 11 | 1/15 | LECTURE 11: MULTI-PHASE FLOW PROBLEMS

CONTENTS

NOTE: MULTI-PHASE FLOW PROBLEMS HAVE BEEN LITTLE EXPLORED; THIS LECTURE PROVIDES THE CONCEPTUAL AND MATHEMATICAL FRAMEWORK.

MPF 11 | 2/15 | PROBLEM STATEMENT, 1

- THE STEAM-GENERATOR PROBLEM (LECTURE 9) WAS ALREADY FOUR-PHASE (STEAM, WATER, METAL, PRIMARY FLUID); IT IS NOT THIS KIND OF MULTI-PHASE PROBLEM THAT IS IN QUESTION HERE.
- WHEN MANY PHASES ARE IN MOTION WITHIN THE SAME SPACE, IPSA REQUIRES (SLIGHT) MODIFICATION.
- WHEN PHASES ARE DISTINGUISHED BY THEIR "LOCATION IN PHASE SPACE", EG BY UPPER AND LOWER PARTICLE-SIZE LIMITS, INTERPHASE MASS TRANSPORT REQUIRES ATTENTION.
- IF MOTION IS ONE-WAY IN PHASE SPACE, ECONOMIES IN COMPUTER STORAGE CAN BE EFFECTED.

MPF 11	3/15	PROBLEM STATEMENT, 2

- TO FIX IDEAS, SUPPOSE THAT THE DENSER MATERIAL CONSISTS OF PARTICLES OF CONTINUOUSLY VARYING SIZE s, DIVIDED INTO PHASES, WITH INDEX k, SO THAT THE k'th PHASE HAS SIZES BETWEEN s_{k-1}, and s_k, ALL s_m VALUES BEING SPECIFIED, WITH $s_0 = 0$.
- LET ALL PHASE-k MATERIAL AT ONE TIME AND LOCATION HAVE THE SAME TEMPERATURE, T_k, AND VELOCITY COMPONENTS U_k, V_k, W_k.
- LET VOLUME FRACTIONS OF THE PHASES BE DENOTED BY R_k.
- THE TASK IS TO COMPUTE R_k, T_k, U_k, V_k, W_k, ALONG WITH r, τ, u, v, w FOR THE LIGHTER PHASE, AT ALL TIMES AND LOCATIONS.
- A 1D SPLIT IS IN QUESTION HERE.

MPF 11	4/15	"IPSA" FOR MULTI-PHASE FLOW: THE DIFFERENTIAL EQUATIONS, 1

- CONTINUITY: $\frac{\partial}{\partial t}(\rho_k R_k) + \mathrm{div}\,(\rho_k R_k \vec{U}_k) = S_k$,

  WHERE $\vec{U}_k$ IS THE VELOCITY VECTOR, WITH COMPONENTS U_k, V_k, W_k.
- CONSERVATION OF SOME PROPERTY ϕ_k:

$$\frac{\partial}{\partial t}(\rho_k R_k \phi_k) + \mathrm{div}\,(\rho_k R_k \vec{U}_k \phi_k) = S_{k\phi}$$

- NOTES: • "DIFFUSION" TERMS HAVE BEEN OMITTED; IF DESIRED THEY CAN BE REGARDED AS CONTAINED WITHIN THE SOURCE TERMS.
  - ϕ_k IS A SYMBOL FOR A GENERAL PROPERTY, AND CAN STAND VARIOUSLY FOR U_k, V_k, W_k, T_k, ETC.
  - THE LIGHTER-PHASE EQUATIONS (FOR r, u, v, w, τ, ETC) MUST ALSO BE SOLVED.

MPF 11	5/15	"IPSA" FOR MULTI-PHASE FLOW: THE DIFFERENTIAL EQUATIONS, 2

- THE JOINT CONTINUITY EQUATION:

$$\frac{\partial}{\partial t}(\rho r + \sum_{\text{all } k} \rho_k R_k) + \mathrm{div}\,(\rho r \vec{u} + \sum_{\text{all } k} \rho_k R_k \vec{U}_k) = 0 .$$

  WHERE ρ IS DENSITY OF LIGHTER PHASE, $\vec{u}$ ITS VELOCITY VECTOR.
- NOTES: o "WEIGHTING" BEFORE ADDITION MAY BE ADVANTAGEOUS.
  - OTHER JOINT EQUATIONS CAN BE FORMED BY ADDITION; FOR EXAMPLE THE JOINT MOMENTUM EQUATION CAN BE EMPLOYED FOR DETERMINATION OF THE PRESSURE FIELD.
  - THE JOINT CONTINUITY EQUATION IS THE MOST IMPORTANT, BECAUSE IT IS THE SOURCE OF PRESSURE CORRECTIONS IN IPSA
  - A FURTHER EQUATION TO BE SATISFIED IS: $r + \sum_{\text{all } k} R_k = 1$.

MPF 11	6/15	"IPSA" FOR MULTI-PHASE FLOW: FINITE-DIFFERENCE EQUATIONS

- THE MAIN DIFFERENCE BETWEEN THE FDE's FOR MULTI-PHASE FLOWS AND THOSE FOR SINGLE- AND TWO-PHASE FLOWS IS THAT THERE ARE MORE OF THEM.
- SOURCE TERMS WILL REQUIRE SPECIAL ATTENTION (SEE SLIDES 10-12)
- THE JOINT CONTINUITY EQUATION TAKES THE FORM:

$$r_P \sum_{\text{all } n} b_n + \sum_{\text{all } k} R_{kP} \sum_{\text{all } n} b_{kn} = \sum_{\text{all } n} (r_n a_n + \sum_{\text{all } k} R_{kn} a_{kn})$$

WHERE: • "WEIGHTING" HAS NOT BEEN EMPLOYED IN THIS SIMPLE EXAMPLE.
- a's AND b's DENOTE INFLOWS AND OUTFLOWS RESPECTIVELY.
- n DENOTES NEIGHBOUR IN SPACE OR TIME;
- "all n" ≡ SUM OVER SIX CELL FACES + TRANSIENT TERM;
- "all k" ≡ SUM OVER ALL DENSE-MATERIAL PHASES.

MPF 11	7/15	"IPSA" FOR MULTI-PHASE FLOW: CORRECTION FORM OF THE JOINT CONTINUITY EQUATION (UNWEIGHTED).

- DEFINITION OF JOINT CONTINUITY ERROR:

  $E \equiv \sum_{all\ n} (r_n a_{n*} + \sum_{all\ k} R_{kn} a_{kn*}) - r_P \sum_{all\ n} b_{n*} - \sum_{all\ k} R_{kP} \sum_{all\ n} b_{kn*}$

  WHERE $*$'s DENOTE COEFFICIENTS BASED ON VELOCITIES WHICH MAY NOT FIT CONTINUITY.

- CORRECTION FORM OF JOINT CONTINUITY EQUATION:

  $r_P \sum_{all\ n} b'_n + \sum_{all\ k} R_{kP} \sum_{all\ n} b'_{kn} - \sum_{all\ n} (r_n a'_n + \sum_{all\ k} R_{kn} a'_{kn}) = E$

  WHERE PRIMED QUANTITIES DENOTE INCREMENTS WHICH, IF ADDED TO THE ASTERISKED QUANTITIES, WILL CAUSE THE JOINT CONTINUITY EQUATION TO BE SATISFIED.

MPF 11	8/15	"IPSA" FOR MULTI-PHASE FLOW: SOLUTION PROCEDURE

- ORDER OF SOLUTION:

  (1) SOLVE INDIVIDUAL CONTINUITY EQUATIONS FOR all k.

  (2) OBTAIN r AS $1 - \sum_{all\ k} R_k$.

  (3) GUESS PRESSURE FIELD p_* (OR OBTAIN IT FROM "SNIP").

  (4) SOLVE FOR u_*, v_*, w_* AND U_{k*}, V_{k*}, W_{k*}, FOR all k.

  (5) COMPUTE E.

  (6) DERIVE p' EQUATION FROM $b' \sim a'$ EQUATION, USING DIFFERENTIALS FROM THE MOMENTUM EQUATIONS.

  (7) SOLVE FOR p', & APPLY VELOCITY & DENSITY CORRECTIONS.

  (8) RETURN TO (1) UNTIL RESIDUALS ARE SUFFICIENTLY SMALL.

MPF 11	9/15	"IPSA" FOR MULTI-PHASE FLOW: DISCUSSION

- THE CHANGES AS COMPARED WITH TWO-PHASE FLOW ARE FEW; AND THEIR NECESSITY IS OBVIOUS.
- THE AMOUNT OF COMPUTATION INCREASES APPROXIMATELY IN PROPORTION TO THE NUMBER OF PHASES; BUT THE COMPLICATION DOES NOT.
- THE COEFFICIENTS IN THE PRESSURE-CORRECTION EQUATION MAY BE SIMPLIFIED IF DESIRED, THE CONTRIBUTIONS OF PHASES PRESENT IN LOW CONCENTRATION BEING NEGLECTED.
- IF "SNIP" IS EMPLOYED AS THE SOURCE OF p_*, THE LIGHT-PHASE MOMENTUM EQUATION IS EASIEST TO USE.

MPF 11	10/15	INTERPHASE MASS TRANSPORT: THE PROBLEM

- DENSE-PHASE MATERIAL ENTERS THE k'th PHASE (size range: $s_{k-1} < s \leq s_k$), AS A CONSEQUENCE OF:

  (1) CONTINUOUS SIZE CHANGE OF PARTICLES BY CONDENSATION ON SMALLER PARTICLES, AND VAPORISATION, SUBLIMATION OF OXIDATION OF LARGER ONES.

  (2) AGGLOMERATION OF SMALLER PARTICLES (EG COLLISION OF DROPLETS).

  (3) BREAK-UP OF LARGER PARTICLES (EG "ATOMISATION" OF DROPLETS).
- DENSE-PHASE MATERIAL LEAVES THE k'th PHASE BY THE SAME MECHANISMS.
- QUANTITATIVE LAWS ARE USUALLY KNOWN FOR GROUP (1) PROCESSES; AND THEY CAN BE PLAUSIBLY ESTIMATED FOR PROCESSES (2) & (3).

MPF 11	11/15	INTERPHASE MASS TRANSPORT: VELOCITY IN PHASE SPACE, $\dot{s}$

- LET THE RATE OF CHANGE OF PARTICLE SIZE WITH TIME BY GROUP (1) PROCESSES BE, $\dot{s}\ \{s, T, \tau, \vec{U}-\vec{u}, \ldots\}$.
- $\dot{s}$ CAN BE REGARDED AS A VELOCITY IN PHASE SPACE. IT CAN BE POSITIVE OR NEGATIVE; AND "UPWIND DIFFERENCING" IS APPROPRIATE TO TRANSFER ACROSS PHASE (IE SIZE-RANGE) BOUNDARIES.
- CONSEQUENTLY, THE SOURCE TERM OF THE k'th PHASE BY GROUP (1) PROCESSES PER UNIT VOLUME IS:

$$(MR)_{k-1}\,[\dot{s}_{k-\frac{1}{2}}] + (MR)_{k+1}\,[-\dot{s}_{k+\frac{1}{2}}] -$$
$$- (MR)_{k}\,([-s_{k-\frac{1}{2}}] + [s_{k+\frac{1}{2}}])$$

- M ALLOWS FOR MASS ENTERING OR LEAVING THE LIGHTER PHASE.

MPF 11	12/15	INTERPHASE MASS TRANSPORT: DISCUSSION

- KNOWLEDGE OF SIZE-CHANGE LAWS, AND OF THOSE OF AGGLOMERATION AND FRAGMENTATION IF PRESENT, THUS ALLOWS ALL SOURCE TERMS IN THE PHASE-CONTINUITY EQUATIONS TO BE ASCRIBED VALUES, IN TERMS OF THE R_k DISTRIBUTION.
- THE SOURCE TERMS REPRESENT THE LAST UNUSUAL FEATURE OF THE MULTI-PHASE FLOW PROBLEM. THE MATHEMATICAL PROBLEM HAS BECOME SOLUBLE.
- OF COURSE, THE REALISM OF THE SOLUTIONS DEPENDS UPON THAT OF THE PRESUMED LAWS OF MASS TRANSPORT; FOR TYPES (2) AND (3), RELIABILITY IS LOW AT PRESENT.

MPF 11	13/15	THE POSSIBILITY OF MARCHING THROUGH PHASE SPACE

- IF THE NUMBER OF PHASES WHICH MUST BE CONSIDERED IS LARGE, AND THE PROCESS IS 3D, COMPUTER-STORAGE REQUIREMENTS MAY BE DIFFICULT TO MEET.
- HOWEVER, IF:
  - THE FLOW IS STEADY,
  - $\dot{s}$ IS ALWAYS ≥ 0, OR ALWAYS ≤ 0,
  - PROCESSES (2) AND (3) ARE NEGLIGIBLE,

  STORAGE IS NEEDED FOR ONLY ONE OF THE PHASES AT A TIME.
- THE SECOND CONDITION IMPLIES: VAPORISATION WITHOUT CONDENSATION; OR VICE VERSA.
- IN THESE CIRCUMSTANCES, STAGE (1) OF THE SOLUTION PROCEDURE OF SLIDE 8 IS CONDUCTED AS A "MARCH THROUGH PHASE SPACE", QUANTITIES NECESSARY FOR SUBSEQUENT STAGES ARE ACCUMULATED FOR LATER USE AT EACH STEP.

MPF 11	14/15	PRESENT STATUS OF MULTI-PHASE FLOW CALCULATIONS

- CALCULATIONS OF MULTI-PHASE FLOWS WITHOUT SLIP BETWEEN THE PHASES HAVE BEEN CONDUCTED FOR MANY YEARS (SPALDING, 1970; GIBSON & MORGAN, 1970; SALA & SPALDING, 1973; RICHTER & QUACK, 1974; ELGHOBASHI ET AL, 1976; MONGIA, 1978; SWITHENBANK ET AL, 1978).
- ALLOWANCE FOR INTERPHASE SLIP IS JUST BEGINNING.
- THE MATHEMATICAL TECHNIQUES ARE NOW AVAILABLE, AND PRELIMINARY POSTULATES ARE AVAILABLE FOR THE INTERPHASE TRANSPORT TERMS.
- EXPERIMENTAL VALIDATION WILL BE REQUIRED.

MPF 11	15/15	THE FUTURE OF MULTI-PHASE FLOW MODELLING

- WHY SHOULD ALL PARTICLES OF THE SAME SIZE RANGE HAVE THE SAME TEMPERATURE? IN A TURBULENT FLOW IT IS NOT LIKELY. THEN A 2D PHASE SPLIT IS NEEDED; AND COMPUTER TIME AND STORAGE REQUIREMENTS BECOME SEVERE.
- WHEN SPLITS WITH RESPECT TO VELOCITY COMPONENTS ARE ALSO ENVISAGED, THE REQUIREMENTS BECOME PROHIBITIVE.
- THEN PROBABLY IT WILL BE NECESSARY TO CONFINE ATTENTION TO THE PHASE-AVERAGE $\bar{U}$, $\bar{V}$, $\bar{W}$ AND TO A SINGLE MEASURE OF DEVIATION FROM EACH: U', V', W'.
- SUCCESS DEPENDS UPON INTRODUCING INTO THE MODEL JUST ENOUGH REFINEMENT, AND NO MORE.

MPF 12	1/30	LECTURE 12: REVIEW, AND FURTHER DEVELOPMENTS

CLASSIFICATION:

1. PHASES WELL SEPARATED
   - DROPLET IMPINGEMENT
   - FILM CONDENSATION
   - "BREAKING" OF A WAVE
   - BUBBLE BREAKAWAY AND RISE
   - "SLUG FLOW" OF GAS AND LIQUID IN A PIPE
   - DRAINAGE OF A TANK UNDER ZERO GRAVITY
2. PHASES WELL INTERMINGLED
   - "BUBBLY FLOW" OF A STEAM-WATER MIXTURE
   - TRANSPORT OF SNOW BY WIND, OR SAND BY WATER
   - FLUIDISED "BED"
3. INTERMEDIATE PHENOMENA
   - ANNULAR FLOW OF STEAM-WATER MIXTURE WITH ENTRAINED DROPLETS
   - WIND-BLOWN SURF

MPF 12	2/30	WHY NUMERICAL COMPUTATION?

WHY COMPUTATION?

- MATHEMATICAL SIMULATION TESTS AND PROMOTES UNDERSTANDING.
- EXPERIMENTAL MEASUREMENTS HAVE WIDER SIGNIFICANCE IF EXTRAPOLATED BY PROVED MATHEMATICAL MODELS.
- EQUIPMENT (CONDENSERS, BOILERS, REACTORS, FURNACES) MAY BE OPTIMISED INEXPENSIVELY.

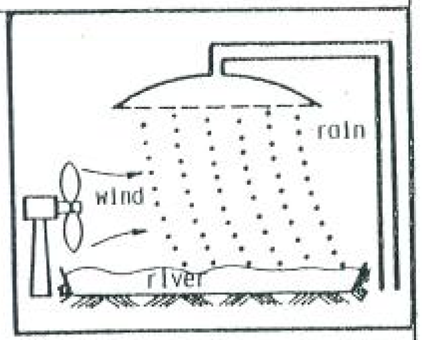
WHY NUMERICAL?

- GOVERNING EQUATIONS ARE NON-LINEAR, AND STRONGLY COUPLED.
- PHENOMENA ARE TWO- AND THREE-DIMENSIONAL.
- CLASSICAL MATHEMATICS CANNOT COPE.

MPF 12 | 3/30 | STATUS OF NUMERICAL-COMPUTATION CAPABILITIES

- SINGLE-PHASE PHENOMENA
  - NO SERIOUS COMPUTATIONAL DIFFICULTIES REMAIN.
  - CORRECT REPRESENTATION OF TURBULENCE REQUIRES RESEARCH.
  - 3D, COMPRESSIBLE, TRANSIENT, REACTING FLOWS REMAIN DIFFICULT.
- WELL-SEPARATED TWO-PHASE PHENOMENA
  - HANDLING PHASE INTERFACE HAS PRESENTED DIFFICULTIES, NOW SOLVED.
- WELL-INTERMINGLED FLOWS
  - WHEN "SLIP" IS ABSENT, THE TWO PHASES ACT AS ONE.
  - COMPUTATION OF "SLIPPING" FLOWS IS WELL UNDER WAY.

MPF 12 | 4/30 | WHY "SLIP-FLOW" COMPUTATIONS ARE NOT STRAIGHTFORWARD



- AT EACH LOCATION, THREE VELOCITY COMPONENTS MUST BE COMPUTED FOR EACH PHASE.
- LAWS OF HEAT, MASS AND MOMENTUM TRANSFER BETWEEN THE PHASES MUST BE CORRECTLY ACCOUNTED FOR.
- VOLUME FRACTIONS VARY FROM 0 TO 1, AND MUST BE COMPUTED FROM THE TWO CONTINUITY EQUATIONS.
- CONFUSION HAS EXISTED IN THE LITERATURE AS TO THE CORRECT FORMULATION, AND THE MATHEMATICAL PROPERTIES OF THE EQUATIONS.

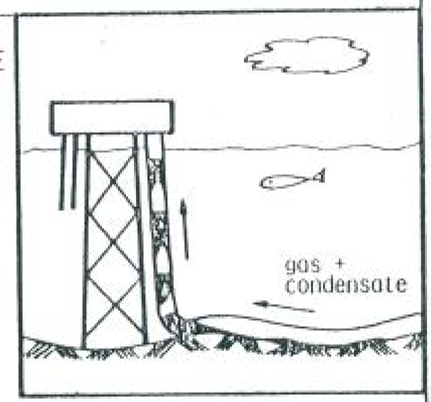
MPF 12 | 5/30 | OUTLINE OF PRESENT LECTURE

- MOTIVE:

  TO EXPLAIN HOW A NUMERICAL MODELLER PROCEEDS.
- METHOD:

  TO DESCRIBE THE CONCEPTION, BIRTH AND PROSPECTS OF A PARTICULAR COMPUTATIONAL MODEL.
- INTENDED EFFECTS:
  - TO SHOW THAT THE BASIC IDEAS ARE EASY TO UNDERSTAND.
  - TO ENCOURAGE APPROPRIATE USE OF NUMERICAL MODELS.
  - TO EXPLAIN WHAT SHOULD AND SHOULD NOT BE EXPECTED OF THEM.

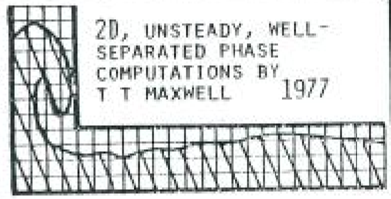
MPF 12 | 6/30 | PRECURSORS, 1: THE UNDER-OCEAN PIPE-LINE PROBLEM



- GAS MUST BE PUMPED OVER LARGE DISTANCES.
- CONDENSATE FORMING IN THE PIPES COLLECTS IN LOWER REGIONS, THEN SURGES UP THE RISER IN "SLUGS".
- THESE MUST BE PREVENTED FROM REACHING (AND DAMAGING) GAS COMPRESSORS; SO LARGE "SLUG CATCHERS" MUST BE BUILT.
- PROBLEMS: WHAT INFLUENCES SLUG SIZE? CAN SIZE BE REDUCED?

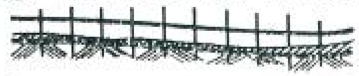
MPF 12	7/30	PRECURSORS, 2: PRELIMINARY STUDIES OF SLUG FORMATION

- BECAUSE 3D COMPUTATIONS ARE EXPENSIVE, A 2D ANALYSIS WAS MADE.
- RESULTS WERE QUALITATIVELY INTERESTING, AND RAISED THE QUESTION: WHAT ARE THE INFLOW CONDITIONS?



- INFLOW TO BEND DEPENDS ON WAVES FORMING IN LONG PIPE.
- 1D STRATIFIED-FLOW ANALYSIS PREDICTS THESE WAVES, ACCURATELY WHERE TESTS ARE POSSIBLE.
- WOULD HORIZONTAL BENDS HELP?

1D, UNSTEADY, TWO-LAYER COMPUTATIONS BY BAGHDADI ET AL (1977).
NB THIS IS 1D FLOW WITH SLIP.

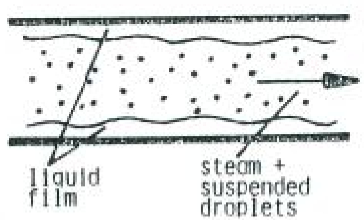


2 VELOCITIES AT EACH SECTION. DEPTH VARIABLE.

MPF 12	8/30	PRECURSORS, 3: STUDIES OF STEAM-WATER FLOW IN PIPES

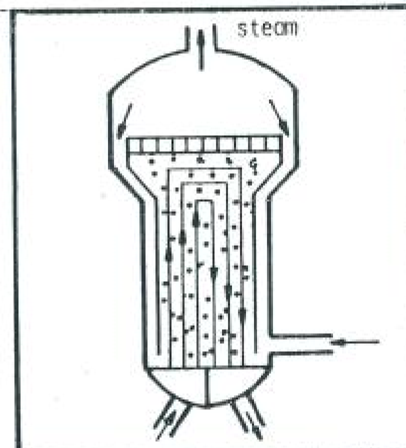
- FLOW OF STEAM AND WATER IS DESCRIBED IN TERMS OF: WATER ANNULUS; STEAM CORE; SUSPENDED DROPLETS.
- "ENTRAINMENT" OCCURS, PARTICULARLY FROM THE SURFACE OF THICK WAVES.
- EXPERIMENTS ON ENTRAINMENT RATES AND DEPOSITION RATES ARE BEING MADE, EG BY HEWITT, HANRATTY, ETC.
- ANALYSIS IS OF STEADY STATES.

Conceptual Model Used in Steady-Flow Analysis



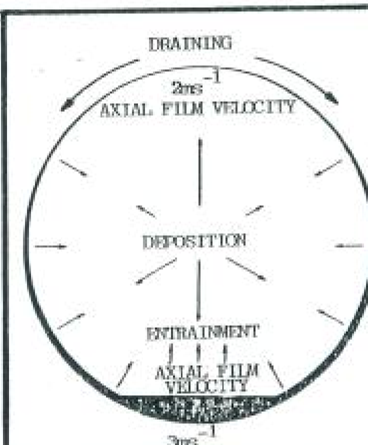
MPF 12	9/30	PRECURSORS, 4: NUMERICAL MODELLING OF 3D FLOW WITH SLIP IN STEAM GENERATORS

- NUMERICAL COMPUTATIONS ARE BEING PERFORMED OF 3D STEADY AND UNSTEADY 2-PHASE FLOWS IN NUCLEAR STEAM GENERATORS.
- FLOW IS WELL INTERMINGLED; BUT SUB-COOLED, 2-PHASE AND SUPERHEATED REGIONS ALL OCCUR.
- COMPUTATIONS ARE NOT INEXPENSIVE.



MPF 12	10/30	PRECURSORS, 5: A CONCEPT OF FILM SHAPE IN STRATIFIED FLOW

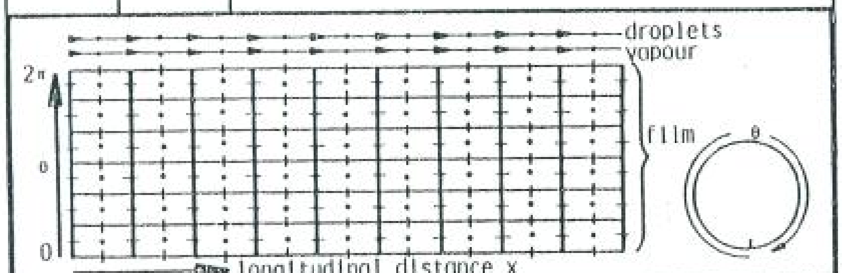
- FISHER AND PEARCE (1978) HAVE PRESENTED A STEADY-STATE, SINGLE-SECTION, ANALYSIS OF FILM DRAINAGE TAKING ACCOUNT OF DEPOSITION, GRAVITY, VISCOSITY IN THE FILM.
- THIS PICTURE IS THE STARTING POINT FOR THE NUMERICAL MODEL NOW TO BE DESCRIBED.



MPF 12	$\frac{11}{30}$	THE NEW MODEL: FILIPA (FOR FILM-IN-PIPE ANALYSER)

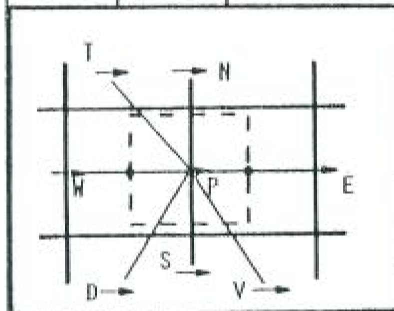
- CHARACTER:
  - 2D TRANSIENT FOR FILM;
  - 1D TRANSIENT FOR CORE;
  - 3-PHASE (FILM, VAPOUR, DROPLETS).
- INDEPENDENT VARIABLES:
  - LONGITUDINAL DISTANCE;
  - PERIPHERAL DISTANCE (ANGLE);
  - TIME.
- DEPENDENT VARIABLES:
  - FILM THICKNESS (2D);
  - TWO FILM-VELOCITY COMPONENTS (2D);
  - VAPOUR VELOCITY (1D);
  - DROPLET VELOCITY AND VOLUME FRACTION (1D);
  - POSSIBLY TEMPERATURES OF FILM (2D), VAPOUR AND DROPLET (1D).

MPF 12	$\frac{12}{30}$	THE NEW MODEL: THE GRID



- FILM THICKNESSES STORED AT •'s.
- FILM VELOCITIES STORED AT -'s AND ı's.
- ALL POINTS IN FILM ARE IN CONTACT WITH VAPOUR STREAM.
- ALL DROPLETS ARE IN CONTACT WITH VAPOUR STREAM.
- (REMINDER): VAPOUR AND DROPLETS HAVE ONE NODE EACH AT ANY x.

MPF 12	$\frac{13}{30}$	THE NEW MODEL: THE FINITE- DOMAIN EQUATION FOR u



P,N,S,E,W ARE POINTS IN FILM.
V REPRESENTS VAPOUR CORE.
D REPRESENTS DROPLETS.
T REPRESENTS TUBE WALL.
P_- REPRESENTS P AT EARLIER TIME.
U IS MOMENTUM SOURCE IN u - DIRECTION (PRESSURE GRADIENT, GRAVITY, CENTRIFUGAL FORCE, WALL FRICTION...)

$$u_P = \frac{a_N u_N + a_S u_S + a_E u_E + a_W u_W + a_{P_-} u_{P_-} + a_V u_V + a_D u_D + U}{a_N + a_S + a_E + a_W + a_{P_-} + a_V + a_D + a_T}$$

MPF 12	$\frac{14}{30}$	THE NEW MODEL: OTHER FINITE-DOMAIN EQUATIONS

- IF ϕ STANDS FOR: v, THE CIRCUMFERENTIAL VELOCITY, OR t, THE FILM THICKNESS, OR h, ITS ENTHALPY, A SIMILAR FDE CAN BE WRITTEN, VIZ:

$$\phi_P = \frac{a_N\phi_N + a_S\phi_S + a_E\phi_E + a_W\phi_W + a_{P_-}\phi_{P_-} + a_V\phi_V + a_D\phi_D + \Phi}{a_N + a_S + a_E + a_W + a_{P_-} + a_V + a_D + a_T} .$$

- FOR DROPLET AND VAPOUR PROPERTIES, ϕ's HAVE SINGLE VALUES FOR AN x VALUE, BUT ARE CONNECTED WITH ALL FILM GRID-NODES AT THAT x. THUS:

$$\phi_P = \frac{a_E\phi_E + a_W\phi_W + a_{P_-}\phi_{P_-} + \Sigma a_1\phi_{1,film} + \Phi}{a_E + a_W + a_{P_-} + \Sigma a_{1,film}} .$$

MPF 12	$\frac{15}{30}$	HOW MANY FINITE- DOMAIN EQUATIONS ARE THERE?

- NUMBER OF EQUATIONS = $n_x \times (n_y \times n_{fv} + 2 \times (n_{fv} - 1))$, WHERE:

  n_x = NUMBER OF LOCATIONS IN LONGITUDINAL DIRECTION,

  n_y = NUMBER OF LOCATIONS IN CIRCUMFERENTIAL DIRECTION,

  n_{fv} = NUMBER OF FILM VARIABLES (EG 4, VIZ 2 VELOCITIES, THICKNESS, ENTHALPY),

  2 = NUMBER OF CORE VALUES AT EACH x,

  -1 REPRESENTS NON-EXISTENCE OF CIRCUMFERENTIAL VELOCITY FOR CORE POINTS.

- SO NUMBER OF EQUATIONS COULD EASILY EQUAL

  $50 \times (10 \times 4 + 2 \times 3) = 2300$, FOR EACH TIME INTERVAL.

MPF 12	$\frac{16}{30}$	WHAT SORT OF FINITE- DOMAIN EQUATIONS ARE THEY?

- THEIR LINEAR APPEARANCE IS DECEPTIVE:
  IN REALITY, THE a's AND ϕ's DEPEND UPON THE ϕ's.

- THEY ARE STRONGLY COUPLED:
  EG SOURCE OF CIRCUMFERENTIAL MOMENTUM DEPENDS ON FILM THICKNESS; AND FILM THICKNESS IS INFLUENCED BY CIRCUMFERENTIAL VELOCITY.

- THEY "SPREAD INFLUENCES" IN THE SPACE DIRECTIONS, BOTH POSITIVELY AND NEGATIVELY, BUT ONLY POSITIVELY IN TIME.

- THEY ARE SIMILAR TO THOSE ENCOUNTERED IN PROBLEMS WHICH HAVE ALREADY BEEN SUCCESSFULLY SOLVED.

MPF 12	$\frac{17}{30}$	WHAT KIND OF SOLUTION PROCEDURE?

- IT WILL "MARCH" FROM EARLY TIMES TO LATE TIMES.

- IT WILL NEED TO BE GIVEN THE STARTING VALUES AT ALL GRID POINTS.

- BOUNDARY (EG EXIT AND ENTRY) VALUES WILL BE NEEDED FOR ALL TIME INTERVALS.

- AT EVERY ADVANCE IN TIME, THE EQUATIONS WILL BE SOLVED BY GUESSING, CHECKING ERRORS, AND IMPROVING THE VALUES.

- A DIGITAL COMPUTER WILL BE NEEDED.

MPF 12	$\frac{18}{30}$	OTHER QUESTIONS AND ANSWERS ABOUT THE SOLUTION PROCEDURE

Q HAVE PROBLEMS OF THIS KIND BEEN SOLVED ALREADY?

A SIMILAR ONES HAVE; BUT NOT PRECISELY OF THIS KIND.

Q WHAT COULD GO WRONG?

A AN ILL-CHOSEN "GUESS-AND-IMPROVE" PROCEDURE MIGHT NOT SUFFICIENTLY DIMINISH ERRORS IN A FINITE TIME; IT MIGHT EVEN ENLARGE THEM.

Q CAN ONE BE SURE THAT A CONVERGENT PROCEDURE EXISTS?

A YES; BECAUSE THE PHENOMENA EXIST.

Q HOW CAN IT BE OBTAINED?

A BY COPYING THE SUCCESSFUL PRACTITIONERS.

MPF 12	$\frac{19}{30}$	PHYSICAL HYPOTHESES EMBODIED IN FILIPA, 1: TRANSPORT THROUGH THE FILM

- BASIC ASSUMPTION:
  PROFILES OF VELOCITY, TURBULENCE PROPERTIES AND TEMPERATURE DEPEND MAINLY ON:
  THICKNESS, FLOW RATE, HEAT FLUX, WALL ROUGHNESS.
- CONSEQUENCES:
  - SHEAR STRESS CAN BE COMPUTED FROM FORMULAE FOR THE "UNIVERSAL VELOCITY PROFILE".
  - HEAT-TRANSFER COEFFICIENTS ARE COMPUTED SIMILARLY.
  - EMPIRICAL AUGMENTATION CAN ACCOUNT FOR WAVINESS.
- NOTE: SUCH ASSUMPTIONS ARE WELL-KNOWN IN CONDENSATION THEORY, AND REPRESENT NO NEW DEPARTURE.

MPF 12	$\frac{20}{30}$	PHYSICAL HYPOTHESES EMBODIED IN FILIPA, 2: DROPLET ~ VAPOUR INTERACTIONS

- BASIC ASSUMPTION (REMINDER):
  DROPLETS ARE SUFFICIENTLY UNIFORM IN SIZE, VELOCITY, TEMPERATURE, FOR A ONE-DIMENSIONAL ANALYSIS TO SUFFICE.
- FRICTION:
  DROPLET DRAG DEPENDS ON REYNOLDS, WEBER NUMBERS IN ACCORDANCE WITH USUAL LAWS.
- HEAT TRANSFER: EITHER
  (I) THERMAL-EQUILIBRIUM PREVAILS BETWEEN STEAM AND WATER;
  (II) HEAT TRANSFER OBEYS USUAL LAWS OF DROPLET VAPORIZATION.
- NOTE: ONCE AGAIN, NO NOVELTY IS INVOLVED HERE.

MPF 12	$\frac{21}{30}$	PHYSICAL HYPOTHESES EMBODIED IN FILIPA, 3: DROPLET ENTRAINMENT AND DEPOSITION

- BASIC ASSUMPTIONS:
  (I) ENTRAINMENT (IE DROPLET FORMATION) DEPENDS ON LOCAL VAPOUR VELOCITY, FILM THICKNESS, THICKNESS GRADIENT, DENSITY DIFFERENCE, ETC.
  (II) ENTRAINED DROPLETS HAVE FINITE LATERAL MOMENTUM, WHICH PERSISTS DURING THEIR FLIGHT.
- ENTRAINMENT LAWS: ENTRAINMENT RATE PER AREA:
  $\dot{m}'' = \rho_{liq}|u_{vap} - u_{liq}| . f[\rho_{gas}(u_{vap} - u_{liq})^2/(\rho_{liq} g t), \rho_{liq}\delta u_{liq}/\nu_{liq}, \ldots$, terms accounting for surface tension, etc].
- DEPOSITION LAWS:
  TO BE WORKED OUT FROM TRAJECTORY CONSIDERATIONS, SUPPLEMENTED BY EXPERIMENTAL DATA.

MPF 12	$\frac{22}{30}$	PHYSICAL HYPOTHESES EMBODIED IN FILIPA, 4: OTHER

- VAPOUR ~ FILM FRICTION LAWS MUST BE ESTIMATED, THE EFFECT OF WAVINESS CAN BE ESTIMATED.
- IF THE VAPOUR CONTAINS A CONDENSABLE AND A NON-CONDENSABLE GAS, A MASS-TRANSFER LAW MUST BE INCLUDED WHICH REPRESENTS THE BLOCKING EFFECT OF THE VAPOUR.
- IF PRESSURE VARIATIONS ARE APPRECIABLE, THE COMPRESSIBILITY OF THE VAPOUR, AND THE SHIFT IN THE SATURATION TEMPERATURE, MUST BE CONSIDERED.
- OTHER HYPOTHESES?
  PROBABLY NOT.

MPF 12	23/30	SOME EXAMPLES OF USE, 1: DESIGN OF UNDER-OCEAN PIPE LINES

- THE PROBLEM:
  TO REDUCE THE SIZE OF THE LARGEST SLUGS OF LIQUID ENTERING THE PUMPING STATION (REF PANEL 6).
- THE IDEA:
  PROVISION OF BENDS IN THE HORIZONTAL PIPE, NEAR THE FOOT OF THE RISER, MIGHT REDUCE THE LENGTH OF WAVES BY PROVIDING AN ADDITIONAL CENTRIFUGAL COMPONENT.
- WHAT FILIPA COULD DO:
  PREDICT THE UNSTEADY TWO-PHASE FLOW PHENOMENA IN THE PIPE, FOR VARIOUS GEOMETRICAL ARRANGEMENTS AND FLOW RATES.
- NOTE: THE DROPLET STREAM IS PROBABLY NOT IMPORTANT IN THIS PROBLEM.

MPF 12	24/30	SOME EXAMPLES OF USE, 2: PREDICTION OF STEADY TWO-PHASE FLOW IN COILED PIPES

- THE PROBLEM:
  WATER ~ LIQUID FLOWS IN HELICAL PIPES CANNOT BE AXI-SYMMETRICAL, BECAUSE LIQUID IS FLUNG TO OUTSIDE OF BEND. HOW CAN PRESSURE DROP/HEAT TRANSFER BE PREDICTED.
- THE IDEA:
  POSTULATE STEADY INLET AND OUTLET FLOWS; NEGLECT GRAVITY; SEEK STEADY-STATE SOLUTIONS.
- WHAT FILIPA COULD DO:
  SOLVE RELEVANT EQUATIONS; PREDICT DISTRIBUTIONS AROUND THE PERIPHERY OF: FILM THICKNESS; SHEAR STRESS; HEAT-TRANSFER COEFFICIENT.
- NOTE: DROPLET FLOW MAY NOT BE IMPORTANT IN THIS CASE.

MPF 12	25/30	EXAMPLES OF USE, 3: STABILITY OF FLOW IN COILED PIPES

- THE PROBLEM:
  WAVES ON FILM SURFACE MAY GROW BECAUSE INCREASED THICKNESS REDUCES CORE AREA, INCREASES VAPOUR VELOCITY, SHEAR STRESS, ETC. STEADY FLOW MAY BE IMPOSSIBLE.
- THE IDEA:
  FOR A VARIETY OF LIQUID/VAPOUR FLOW RATES, PIPE/COIL RATIOS, ETC, INVESTIGATE CONSEQUENCE OF IMPOSING A PERTURBATION IN FILM THICKNESS.
- WHAT FILIPA COULD DO:
  SOLVE THE TRANSIENT-FLOW EQUATIONS WITH A FINE-ENOUGH GRID TO ENSURE ACCURACY, AND DETERMINE UNDER WHAT CONDITIONS THE PERTURBATION DIES AWAY.
- WILL THE RESULT BE CREDIBLE?
  YES, IF GRID-INDEPENDENCE CAN BE ATTAINED.

MPF 12	26/30	EXAMPLES OF USES, 4: THE PIPE-BREAK PROBLEM (EG LOCA)

- THE PROBLEM:
  A BREAK OCCURS IN A PIPE CARRYING PRESSURISED WATER; STEAM "FLASHES", AND THE MIXTURE FLOWS OUT THROUGH THE BREAK. FIND HOW LARGE ARE EFFECTS ELSEWHERE IN THE SYSTEM, AND HOW SOON THEY OCCUR.
- THE IDEA:
  COMPUTER CODES TREATING THE FLOW AS 1D EXIST; BUT EFFECTS OF GRAVITY, AND BENDS, ARE INEVITABLY NEGLECTED. 3D TRANSIENT CODES ALSO EXIST; BUT THEY ARE TOO EXPENSIVE.
- WHAT FILIPA COULD DO:
  REPEAT SOME 1D CALCULATIONS TO ENSURE CONFORMITY; THEN INVESTIGATE THE INFLUENCES NEGLECTED BY THEM, VIZ PRESENCE OF FILM AND DROPLETS; GRAVITY; BENDS; HEAT TRANSFER.

MPF 12	27/30	EXAMPLES OF USE, 5: BURN-OUT, ETC

- THE PROBLEM:
  A STEAM-WATER MIXTURE FLOWS IN A PIPE, THE WALLS OF WHICH ARE HEATED. WHEN SUFFICIENT OF THE WATER HAS BEEN EVAPORATED, THE FILM MAY DISAPPEAR ENTIRELY, PERMITTING OVER-HEATING OF THE METAL.
- THE IDEA:
  BURN-OUT IS MOST PROBABLE AT THE TOP, IF THE PIPE IS APPROXIMATELY HORIZONTAL, OR AT THE INSIDE OF A BEND. SUCH EFFECTS NEED TO BE PREDICTED.
- WHAT FILIPA COULD DO:
  IMPROVE UPON EXISTING ANALYSES BY TAKING ACCOUNT OF THE ABOVE NON-UNIFORMITIES.
- NOTE:
  THE NEED FOR SUCH IMPROVEMENT IS <u>URGENT</u>.

MPF 12	28/30	EXTENSION OF FILIPA FLOW IN ROD AND TUBE BUNDLES

- THE PROBLEM:
  IN NUCLEAR REACTORS AND STEAM GENERATORS, TWO-PHASE FLOW OCCURS OUTSIDE RODS AND TUBES; BUT A VAPOUR CORE, CONTAINING DROPLETS, CAN STILL BE DISTINGUISHED.
- THE IDEA:
  EXTEND THE FILIPA ANALYSIS TO MORE COMPLEX "WALL" SHAPES; HENCE PREDICT FLOW REGIMES, BURN-OUT, ETC IN FLOWS THROUGH BUNDLES.
- WHAT FILOMENA COULD DO:
  SOLVE THE ABOVE AND SIMILAR PROBLEMS MAINLY BY USE OF A DIFFERENTLY SHAPED GRID.
- NOTE: FILOMENA STANDS FOR:
  FILM-OUTSIDE-MULTIPLE-ELEMENT ANALYSER.

MPF 12	29/30	REMARKS ON COMPUTER-MODEL CREATION

- COMPUTER MODELS OF PHYSICAL PHENOMENA ARE USEFULLY REGARDED AS ARTISTIC PRODUCTIONS.
- THE MORE EXTENSIVELY AND REALISTICALLY THEY ARE IMAGINED BEFORE CONSTRUCTION BEGINS, THE MORE SUCCESSFUL THEY WILL BE.
- DRAWING UP THE SPECIFICATION IS AS MUCH A CREATIVE ACT AS PROVIDING THE FINISHED COMPUTER CODE.
- A SINGLE SPECIFICATION CAN BE (NOMINALLY) SATISFIED IN MANY WAYS; BUT GOOD "FORM" BRINGS GREAT EXTRA BENEFITS.
- EVEN THE NAME OF THE CODE HAS AN INFLUENCE. (SEE APPENDIX).

MPF 12	30/30	CLOSING REMARKS

- NUMERICAL MODELS OF FLUID-FLOW AND HEAT-TRANSFER PROCESSES ARE WELL ESTABLISHED FOR HOMOGENEOUS PHASES; THEY ARE NOW UNDER ACTIVE DEVELOPMENT FOR SEPARATED AND INTERSPERSED PHASES.
- NUMERICAL MODELS SHOULD NOT INCLUDE EVERYTHING ABOUT THE PHENOMENA MODELLED, BUT JUST ENOUGH.
- MANY TWO-PHASE PHENOMENA OF IMPORTANCE AWAIT EVEN THEIR FIRST NUMERICAL STUDY; THEY ARE VIRGIN TERRITORY.
- MODEL DEVELOPMENT COMBINES TECHNOLOGY WITH ARTISTRY; WHAT HAS BEEN DONE WELL ONCE, CAN STILL BE DONE MUCH BETTER.

GLOSSARY OF FORTRAN VARIABLE NAMES

FORTRAN VARIABLE	MEANING		SYMBOL
A	TDMA coefficient		
ABS	Absolute value of		
ADDDP	An increment added to pressure increment DP		
ADJ	Subroutine		
ADJ2P	Subroutine		
ADPEI	Area of cell divided by PEI		
ADPEIU	Upstream value of ADPEI		
ADUCTD	Downstream duct area		
AFLOWD	Downstream flow area		
AFLOWU	Upstream flow area		
AINC	An increment in R2 when mixing of 2nd phase considered		
AK	Von Karman constant		
AL	Ratio of velocity components v/u for 1st phase		
ALEX	Index of CF	(=1)	
ALIN	Index of CF	(=5)	
ALOG	Napierian logarithms		ln.
ALU	Upstream value of AL		
AL2	AL for 2nd phase		
AL2EX	Index of CF	(=9)	
AL2IN	Index of CF	(=13)	
AL2U	Upstream value of AL2		
AM	Dimensionless mass transfer rate		
AMAX1	Largest value of		
AMIN1	Smallest value of		
AMRE	AM x Reynolds number		
AMRESQ	AMRE squared		
AREA	Cell area		
AREAU	Upstream value of AREA		

FORTRAN VARIABLE	MEANING		SYMBOL
ARG	Argument of logarithm		
ARGMIN	Smallest value of ARG		
ASTORE	Storage		
AVI	Storage		
B	TDMA coefficient		
BEE	Exponent		
BIG	A large number	(=1E2Ø)	
BLANK	A printer space		
BLOC	Subroutine, BLOCK DATA		
BOM	Distance between ω interface		Ω
BP	Stream function coefficient		Ψ
BPE	BP at E boundary		Ψ_E
BPI	BP at I boundary		Ψ_I
BPLAST	Last value of BP		
BVI	Storage		
BVIP1	Storage		
C	TDMA coefficient		
CALEX	Coeff. of CUBIC for AL at E, called by ALEX		
CALIN	Coeff. of CUBIC for AL at I, called by ALIN		
CAL2EX	Coeff. of CUBIC for AL2 at E, Called by AL2EX		
CAL2IN	Coeff. of CUBIC for AL2 at I, called by AL2IN		
CCSALF	Coeff. of CUBIC for CSALFA, called by CSALF		
CF	Coeff. of arithmetic function CUBIC		
CFHEX	Coeff. of CUBIC for H at E, called by FHEX		
CFHIN	Coeff. of CUBIC for H at I, called by FHIN		
CF2EX	Coeff. of CUBIC for H2 at E, called by F2EX		
CF2IN	Coeff. of CUBIC for H2 at I, called by F2IN		

FORTRAN VARIABLE	MEANING	SYMBOL
CHEX	Coeff. of CUBIC for height at E, called by HEX	
CHIN	Coeff. of CUBIC for height at I, called by HIN	
COMP2P	Subroutine	
CON	Lateral convection quantity	
CONST	Constant	
CONST1	Constant	
CONST2	Constant	
CONST3	Constant	
COSD2	CSALFA divided by 2	$\cos\alpha/2$
CP1	Specific heat of 1st phase	
CP2	Specific heat of 2nd phase	
CRADEX	Coeff. of CUBIC for radius at E, called by RADEX	
CRADIN	Coeff. of CUBIC for radius at I, called by RADIN	
CROSS	A printer symbol "x"	
CR2EX	Coeff. of CUBIC for R2 at E, called by R2EX	
CR2IN	Coeff. of CUBIC for R2 at I, called by R2IN	
CSALF	Index of CF (=17)	
CSALFA	Cosine α	$\cos\alpha$
CSTORE	Storage	
CTANEX	Coeff. of CUBIC for tangent of E, called by TANEX	
CTANIN	Coeff. of CUBIC for tangent of I, called by TANIN	
CUBIC	Arithmetic function used in MAIN	
CUEX	Coeff. of CUBIC for U at E, called by UEX	
CUIN	Coeff. of CUBIC for U at I, called by UIN	
CU2EX	Coeff. of CUBIC for U2 at E, called by U2EX	
CU2IN	Coeff. of CUBIC for U2 at I, called by U2IN	

FORTRAN VARIABLE	MEANING	SYMBOL
D	TDMA Coefficient	
DA	Area increment	
DADP	Rate of change in area with pressure	
DA1	1st dimensionless area error	
DA2	2nd dimensionless area error	
DDP	Change in pressure increment DP	
DFE	F-difference at E boundary	
DHTDR	Variation of height with R2 in stratified flow	
DIF	Diffusion quantity	
DIFR2	Constant to change R2, gives provision for mixing of 2nd phase	
DIFU	Diffusion quantity related to velocity U	
DIGIT	Number printed beside X-axis	
DISTAN	Section of subroutine COMP	
DOT	A printer symbol "."	
DP	Pressure increment	
DPMAX	Maximum DP	
DPLAST	Last value of DP	
DUCT	Confined flow when set TRUE	
DUNM1	U difference at (N-1)th node	$(\delta u)_{n-1}$
DU2	U difference at 2nd node	$(\delta u)_2$
DVI	Storage	
DX	Forward step size	δx
DXDPEI	DX÷PEI	$\delta x/(\Psi_E-\Psi_I)$
DXLAST	Last value of DX	
DXPEI	DX x PEI	$\delta x(\Psi_E-\Psi_I)$
DXSQ	DX squared	δx^2
EF	Dimensionless pressure gradient	
EMU	Effective viscosity	
EMU1	Viscosity of 1st phase	μ_1

FORTRAN VARIABLE	MEANING	SYMBOL
ENT	Entrainment quantity	
ER	EWALL x Reynolds number	
ERRMAX	Maximum error in R2	
ERROR	Convergence monitor (R2-R2LAST)	
EWALL	Constant in Wall function	
EXP	Exponential function	
EXPMRE	Expression in laminar wall function	
F	General variable,	ϕ
FDIFE	F increment at E boundary	$\delta\phi_E$
FDIFE	F Increment at I boundary	$\delta\phi_I$
FHEX	Index of CF (=21)	
FHIN	Index of CF (=25)	
FIP	Interphase friction factor	
FIPH	Under-relaxation factor	$f_{i+\frac{1}{2}}$
FLOAT	Change integer variable to real variable	
FLUX	Mass flux	
FLUX2	Mass flux of 2nd phase	
FLUXN	Lateral mass flux, north	
FLUXS	Lateral mass flux, south	
FLUXW	Mass flux from upstream, west	
FRA	Step size ÷ layer width	
FRE	Step x Reynolds number	
F2EX	Index of CF (=29)	
F2IN	Index of CF (=33)	
GRAVX	Gravitational term in x direction	g_x
GRAVY	Gravitational term in Y direction	g_y
GRID	Section of subroutine COMP	
H	Stagnation enthalpy of 1st phase	$\bar{h}$
HCON	CON/2	
HCONI	Constant	

FORTRAN VARIABLE	MEANING	SYMBOL
HCON2	Constant	
HCONDF	HCON difference	
HCONI	HCON at I boundary	
HCOS	Cosine α ÷ 2	$\cos\alpha/2$
HDX	DX ÷ 2	$\delta x/2$
HEAT	Heat transfer considered when set TRUE	
HEXD	Downstream value of height of E boundary	
HEX	Index of CF (=37)	
HIN	Index of CF (=41)	
HIND	Downstream value of height of I boundary	
HINU	Upstream value of height of I boundary	
HIP	Interphase heat transfer factor	
HOMDFE	Half of ω difference at E	
HOMFI	Half of ω difference at I	
HPEI	Half of PEI	$\frac{1}{2}(\Psi_E-\Psi_I)$
HPEIU	Upstream value of HPEI	
HU	Upstream value of H	
H2	H of 2nd phase	
H2U	Upstream value of H2	
I	Index	
IBEX	Index for type of condition at E boundary	
IBIN	Index for type of condition at I boundary	
IDASH	Index used in TDMA	
IDIME	Dimension for array in PLOT	
IDIMF	Dimension for 1D F array in COMP	
IDJ	Used to compute 1D F array indices	
IFIN	Index triggering finish of computation	
IFIX	Integer value with truncation	
IJ	Index of 1D F array equivalent to (I,J)	

FORTRAN VARIABLE	MEANING	SYMBOL
ILDIM	Variable dimension for longitudinal plot	
ILPLOT	Index to obtain longitudinal plot	
IM	Index in PLOT	
IM1	I-1	
IMAX	Number of values to be plotted	
INIT	Section in subroutine COMP	
INJ	Index of 1D F array equivalent to (N,J)	
INMIJ	Index of 1D F array equivalent to (NMI,J)	
INPUT	Subroutine	
IP1	I+1	
IPRINT	Index to control type of print-out required	
IRUN	Index to identify a particular computer run	
ISTEP	Counter of forward step	
IT	Index	
ITDIM	Variable dimension for cross stream plot	
ITER	Iteration counter	
ITEST	Trigger for test output	
ITPLOT	Index to obtain cross stream plot	
IX	Index in PLOT	
IY	Index in PLOT	
I1	I value for WALL	
I2	2nd I value away from WALL	
I3	3rd I value away from WALL	
J	Index	
JAL	F - array index for AL (=2)	
JALU	F - array index for ALU (=9)	
JAL2	F - array index for AL2 (=5)	
JAL2U	F - array index for AL2U (=12)	
JAPEIU	F-array index for APEIU (=18)	
JDIME	Dimension for array in PLOT	

FORTRAN VARIABLE	MEANING	SYMBOL
JERR	F - array index for ERROR (=21)	
JH	F - array index for H (=1)	
JHU	F - array index for HU (=8)	
JH2	F - array index for H2 (=4)	
JH2U	F - array index for H2U (=Ø11)	
JJ	Index	
JLDIM	Variable dimension for longitudinal plot	
JM	Index in PLOT	
IMAX	Number of curves to be plotted	
JP	F - array index for P (=15)	
JR2	F - array index for R2 (=6)	
JR2LST	F - array index for R2LAST (=19)	
JR2U	F - array index for R2 (=13)	
JTDIM	Variable dimension for cross stream plot	
JTEE	F - array index for TEE (=14)	
JTEST	Index for extra test printout	
JTEST1	Index for extra test printout for coeff. in COMP2P	
JUSTEX	Index for boundary condition change at E	
JUSTIN	Index for boundary condition change at I	
JUU	F - array index for UU (=7)	
JU2	F - array index for U2 (=3)	
JU2U	F - array index for U2U (=10)	
JYINT	F - array index for YINT (=16)	
JYINTU	f - array index for YINTU (=17)	
JYU	F - array index for YU (=2Ø)	
J1	Index in INPUT	
J2	Index in INPUT	
K	Index	
KASE	Index denoting problem type	
KEX	Index denoting type of E boundary	

FORTRAN VARIABLE	MEANING	SYMBOL
KIN	Index denoting type of I boundary	
KIND	Index denoting problem type	
KOUT	Number of cross stream variable output	
KRAD	Index denoting geometry type	
KSOURCE	Index denoting source term	
KWALL	Index denoting E or I boundary	
KX	Index	
KY	Index	
L	Index	
LAB	Labels for cross stream profiles	
LASTEP	Maximum value of ISTEP	
M	Index	
MAIN	Main program	
METHOD	Index for R2 correction method in ADJ2P	
MID	Index, N/2	
MINØ	Smallest integer value of	
MOD	Remainder of division	
MODEL	Indicator of transport-process type	
MOMSOU	Index for mementum equation source term	
N	Number of grid points across flow domain	
NEWPR	Index denoting new RECPR	
NEXT	Assign variable	
NF	Number of dependent variables excluding U	
NIT	Number of iterations	
NITER	Maximum number of iterations for each ISTEP	
NM1	N-1	
NM2	N-2	
NM3	N-3	

FORTRAN VARIABLE	MEANING	SYMBOL
NOVEL	No velocity index	
NPLOT	Number of ISTEP after which plots are printed	
NPROF	Number of ISTEP after which profiles are printed	
NSTAT	Number of ISTEP after which station variables are printed	
NX	Index	
NYL	Number of variables for longitudinal plot	
NYT	Number of variables for transverse plot	
OM	Dimensionless stream function	ω
OMDIF	ω difference	$(\omega_i-\omega_{i-1})$
OME	OMDIF at E boundary	
OMI	OMDIF at I boundary	
OMINT	ω for cell interfaces	
OMPOW	Exponent for power law distribution of ω	
OUT	Used for output of cross stream variable profiles	
OUTP	Subroutine	
OUT1	Output from WALL	
OUT2	Output from WALL	
P	Pressure, cross stream profile	
PBP	Logical index for point-by-point algorithm in COMP2P	
PDIFM	Pressure correction term	
PDIFP	Pressure correction term	
PEI	$(\Psi_E-\Psi_I)$ for 1st phase	
PEID	PEI $\div$ DX	
PEIU	Upstream value of PEI	
PEIUD	Upstream value of PEID	
PHYS	Subroutine	
PHYSF	Section of subroutine PHYS	
PHYSU	Section of subroutine PHYS	
PJAY	Jayatillaka's P function	

FORTRAN VARIABLE	MEANING	SYMBOL
PLOTS	Subroutine	
PRESS	Longitudinal pressure	
PRL	Laminar Prandtl number	
PRRAT	Prandtl number ratio	
PR1	Prandtl/Schmidt number for 1st phase	
PR2	Prandtl/Schmidt number for 2nd phase	
PSIE	Stream function at E boundary for for 1st phase	Ψ_E
PSIEU	Upstream value of PSIE	
PSII	Stream function of 1st phase at I boundary	Ψ_I
PSIIU	Upstream value of PSII	
R	Radius	
RADEX	Index of CF (=45)	
RADIN	Index of CF (=49)	
RAT	Ratio of ρur	
RATIO	Ratio	
RE	Reynolds number	
RECOV	Recovery factor	
RECPR	Reciprocal of Prandtl number	
RECPRT	Reciprocal of turbulence Prandtl number	
RECPR1	RECPR for 1st phase	
RECPR2	RECPR for 2nd phase	
RECRU	Reciprocal of density-velocity product	$1/\rho u$
RECR1	Reciprocal of radius of 1st grid node	
RECYDF	Reciprocal of Y difference	$1/\delta y$
RELAX	Under-relaxation for R2 correction	
RHO	Density	
RHOGX	$(\rho 2-\rho 1)\ g_x$	
RHOGY	$(\rho 2-\rho 1)\ g_y$	
RHOREF	Reference density	

FORTRAN VARIABLE	MEANING	SYMBOL
RHOUN	Density-velocity product at node N	$(\rho u)_N$
RHOUSQ	Density-velocity square product	ρu^2
RHO1	Density of 1st phase	$\rho 1$
RHO2	Density of 2nd phase	$\rho 2$
RH1PH	Density of 1st phase at cell interface	$\rho_{i+\frac{1}{2}}$
R1PH	Radius at cell interface	$r_{i+\frac{1}{2}}$
RJTOTE	Radius x total flux at E boundary	$(rJ_{tot,\phi})_E$
RJTOTI	Radius x total flux at I boundary	$(rJ_{tot,\phi})_I$
RM	Radius at cell interface	$r_{i+\frac{1}{2}}$
RMDIF	Net entrainment	
RME	Radius x negative of entrainment at E boundary	$(r\dot{m}'')_E$
RMI	Radius x entrainment at I boundary	$(r\dot{m}'')_I$
RN	Radius at cell interface	
RPRLST	Last value of RECPR	
RREF	Reference radius	
RRUREF	Reference radius x density-velocity product	
RUREF	Density-velocity product	
R1CUB	Radius at 1st node to the power 3	r_1^3
R1D2	Radius at 1st node divided by 2	$r_1/2$
R1D2SQ	R1D2 squared	$(r_1/2)^2$
R2	Volume fraction of 2nd phase	
R2EX	Index of CF (=53)	
R2HIGH	Upper limit of R2	
R2I	R2 at i'th node	
R2IN	Index of CF (=57)	
R2IPH	R2 at cell interface	$R2_{i+\frac{1}{2}}$
R2LAST	Last value of R2	
R2LOW	Lower limit of R2	
R2M	R2 at cell interface	$R2_{i+\frac{1}{2}}$
R2N	R2 at cell interface	$R2_{i+\frac{1}{2}}$
R2U	Upstream value of R2	

FORTRAN VARIABLE	MEANING	SYMBOL
R2UM	R2U at cell interface	
S	Scaling factor in PLOT	
S	Friction factor of Prandtl number in WALL	
SAU	Average value of S in WALL	
SHALF	Square root of S in WALL	
SHALF1	Square root of S in WALL	
SI	Source term	
SINM1	Stored value of SI(NM1)	
SIP	2nd component of Source term	
SI2	Stored value of SI(2)	
SLOC	Local value of S in WALL	
SNALFA	Sine α	
SOLVE	Section of subroutine COMP	
SQRT	Square root of	√
SRE	S x Reynolds number in WALL	
STANE	Stanton number at E boundary	
STANI	Stanton number at I boundary	
STORE	Storage	
STRAT	Logical variable for stratified flow treatment	
SUM	Register for summation	
S1	Storage	
S2	Storage	
S3	Storage	
S4	Storage	
S5	Storage	
S6	Storage	
T	TDMA temporary storage	
TANEX	Index of CF (=61)	
TANI	Tangent of angle at I boundary to x axis	
TANIN	Index of CF (=65)	
TAUE	Shear Stress at E boundary	τ_E
TAUI	Shear Stress at I boundary	τ_I

FORTRAN VARIABLE	MEANING	SYMBOL
TE	Transport coefficient at E boundary	
TEE	Tangent of constant ω line	
TEF	Transport coefficient at E boundary	
TERM1	Longitudinal mass flux term in pressure equation	
TERM2	Longitudinal mass flux term in pressure equation	
TERM3	Lateral mass flux term in pressure equation	
TERM4	Lateral mass flux term in pressure equation	
TERM5	Interphase friction term in pressure equation	
TI	Transport coefficient at I boundary	
TIF	Transport coefficient at I boundary	
TINY	Small number (=1E-1Ø)	
TRNSNT	Logical variable for unsteady flow option	
TWDCOS	2 ÷ cosine α	$2/\cos\alpha$
U	x - direction velocity of 1st phase	
UED	Free stream velocity at E boundary	
UEX	Index of CF (=69)	
UIN	Index of CF (=73)	
UM	U at cell interface	$U_{i+\frac{1}{2}}$
UREF	Reference velocity	
URFAC	Under-relaxation factor for R2 in ADJ2P	
UU	Upstream value of U	
U1IPH	U at cell interface	$U_{i+\frac{1}{2}}$
U2	x - direction velocity of 2nd phase	
U2EX	Index of CF (=77)	
U2IN	Index of CF (=81)	
U2IPH	U2 at cell interface	$U2_{i+\frac{1}{2}}$
U2M	U2 at cell interface	$U2_{i+\frac{1}{2}}$
U2N	U2 at cell interface	$U2_{i+\frac{1}{2}}$

FORTRAN VARIABLE	MEANING	SYMBOL
U2U	Upstream value of U2	
U2UM	U2U at cell interface	$U2U_{i+\frac{1}{2}}$
VOL	Volume of cell	
VREF	Reference viscosity	
WALL	Subroutine	
X	Abscissa x in PLOT	
XAXIS	Label on abscissa x	
XD	Downstream distance of x	
XHEXØ	Value for which HEX takes effect	
XHINØ	Value for which HIN takes effect	
XLAXIS	Label for abscissa in longitudinal plot	
XLPLOT	Downstream distance array for longitudinal plot	
XMAX	Maximum X in PLOT	
XMIN	Minimum X in PLOT	
XR	Scaling variable	
XSIN	x times sine α	$x\sin\alpha$
XSIZE	Scaling factor for printer page	
XTAXIS	Label for abscissa in cross stream plot	
XTPLOT	Cross stream distance array for transverse plot	
XU	Longitudinal distance in steady flow or time in unsteady flow	
XULAST	Largest value of XU	
Y	Cross stream distance	
Y	Plotted ordinate values in PLOT	
YAXIS	Labels for plotted values	
YE	Distance of last cell	
YI	Distance of first cell	
YINT	Y for cell interface	
YINTU	Upstream value of YINT	

FORTRAN VARIABLE	MEANING	SYMBOL
YL	Number printed beside Y-axis	
YLAXIS	Labels for ordinate of plot	
YLPLOT	Values to be plotted	
YMAX	Maximum Y in PLOT	
YMIN	Minimum Y in PLOT	
YN	Distance across flow	
YR	Scaling variable	
YREF	Distance across the couette flow	
YSIZE	Scaling factor for printer page	
YTAXIS	Labels for ordinate of plot	
YTPLOT	Values to be plotted	
YU	Upstream value of Y	

PCH 1980	1/30	3rd INTERNATIONAL CONFERENCE ON PHYSICO CHEMICAL HYDRODYNAMICS MADRID, SPAIN.

NUMERICAL COMPUTATION OF FLOWS OF TWO PHASES SEPARATED BY A MOVING INTERFACE

by

Prof D Brian Spalding

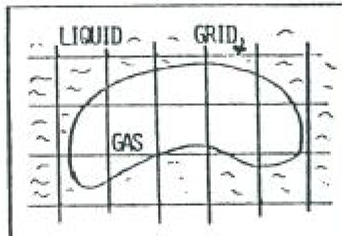
Imperial College
London

March
1980

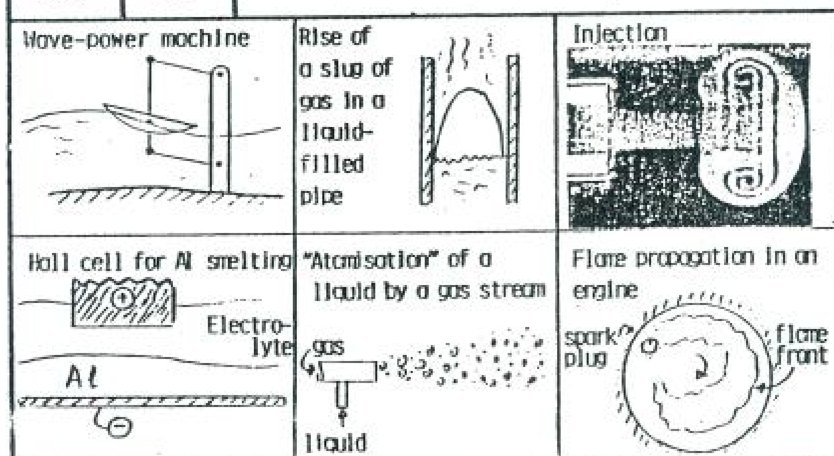
PCH 1980	2/30	CONTENTS

PCH 1980	3/30	THE PROBLEM: ITS NATURE AND SPECIAL DIFFICULTY.

- The task is to predict numerically such phenomena as:-
  - the rise of a gas bubble through a liquid;
  - the injection of one liquid into another;
  - gravity and capillarity waves at liquid-liquid interfaces;
  - propagation of thin reaction fronts (flames) through gases.
- Motive: Numerical prediction procedures permit account for compressibility, temperature-dependence of properties and other complexities.
- Special difficulty: • The interface entails large property variations over distances smaller than inter-nodal ones; • Therefore the usual interpolation procedures do not apply.

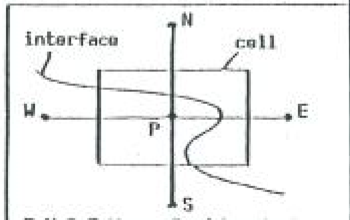


PCH 1980	4/30	SOME PHENOMENA OF PRACTICAL INTEREST



PCH 1980	5/30	FURTHER DETAILS OF THE SPECIAL DIFFICULTY, 1.

- Numerical solution procedures solve algebraic equations connecting dependent variables (ϕ's) at neighbouring grid points.
  - Example: $a_P\phi_P = a_N\phi_N + a_S\phi_S + a_E\phi_E + a_W\phi_W + a_{P-}\phi_{P-} + b$ where a's express effects of transience, flow, diffusion; b is a source.
  - Origin of a's and b: Integration of relevant partial differential equation over cell boundary or volume.
  - Interpolation formulae for ϕ etc: these are needed because values are otherwise known only at grid nodes.



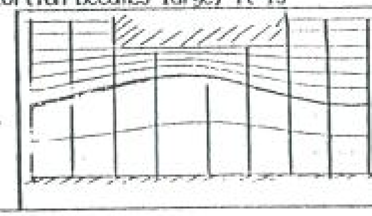
P,N,S,E,W are "grid nodes"

PCH 1980	6/30	FURTHER DETAILS OF THE SPECIAL DIFFICULTY, 2.

- Common interpolation formulae:
  - $\phi = \phi_P$ throughout cell (used in volume integrals);
  - $\phi = \frac{1}{2}(\phi_P + \phi_N)$ along north boundary (used in some boundary integrals, to give a's);
  - $\phi = \phi_P$ or ϕ_N along north boundary according to direction of velocity (implied by some a- formulae).
- Objections to use for problems with interfaces:
  - no account is taken of where interface lies, or of its shape;
  - grid-node values have ceased to be even approximately indicative of "average" values within cell or at boundary.

PCH 1980	7/30	METHODS OF SOLUTION, 1, GRID DISTORTION

- Nature of the method: The grid is distorted. In one or more directions, as the fluids move, so that the interface remains at a fixed location in the grid.
- Advantage: The interpolation problem has been reduced to normal dimensions.
- Disadvantages: • The repeated recomputation of grid dimensions is mildly troublesome. • When the distortion becomes large, it is necessary to modify the formulae employed for the a's and b's etc.
- Limitation: The method fails when interfaces become strongly contorted.
- An application: The Hall-Cell aluminium smelter.



PCH 1980	8/30	METHODS OF SOLUTION, 2; INTERFACE TRACKING.

- Nature of the method: The interface is supposed to carry with it a distributed set of particles, the co-ordinates of which are computed by integrating:

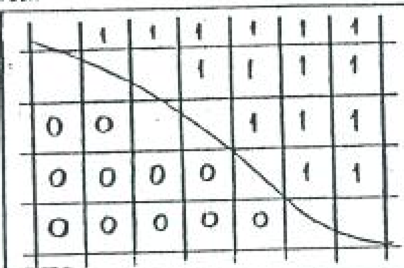
  $$\frac{d\vec{x}_p}{dt} = \vec{u}_p, \text{ where } \vec{x}_p, \vec{u}_p \text{ are particle position and velocity.}$$

- Advantage: If enough particles are employed, the interface can be located with great precision; this permits appropriate interpolation formulae to be devised.
- Disadvantages: • The additional computing time must be provided.
  - Great care is needed to reduce errors arising from:
    (1) deduction of $\vec{u}_p$ by interpolation between grid-point values
    (2) imprecise deductions from interface-location data.

PCH 1980	9/30	METHODS OF SOLUTION, 3; SOLUTION OF A FINITE-DIFFERENCE EQUATION FOR THE VOLUME FRACTION, r.

- Nature of the method: • The interface position is regarded as adequately described by assignment of a volume fraction to one of the phases in each cell, r say. • r is determined for each cell by a phase-mass-balance equation:

  $(r_p \rho_p - r_{p-} \rho_{p-})V/\delta t$ = sum of inflow rate minus outflow rate for all cell walls, with upwind (or other) rule being applied to permit computation of in- and out-flow rates.



PCH 1980	10/30	METHODS OF SOLUTION, 3; THE r-EQUATION METHOD, CONTINUED.

- Advantages: • Exact conservation of both phases is ensured (which is difficult with particle tracking). • Computational expense is small.
- Disadvantages: • No information can be gained (or retained, if provided) about the smaller-than-cell-scale structure of the interface.
- A fine grid is therefore needed so as to preserve precision.
- Unless specially combated by donor-acceptor techniques, "false diffusion" causes further loss of definition.

PCH 1980	11/30	METHODS OF SOLUTION, 4; USE OF A TWO-FLUID MODEL

- Nature of the method: • Two velocity fields, $\vec{u}$ and $\vec{v}$ are computed, one for each phase; and phase-conservation equations (of various kinds) are used for r and R.
- Advantages: • The "lateral slip" of one phase relative to the other in the interface region can be allowed for. • Should the interface break into smaller-than-grid-scale elements, the subsequent behaviour can be represented.
- Disadvantages: • More computer time and storage are needed than for method 3. • Unless special measures are employed, akin to particle tracking, the interface position is 'fuzzy' (as in method 3).

PCH 1980	12/30	METHODS OF SOLUTION, CONCLUDING REMARKS.

- Grid distortion is the best, when it is applicable.
- Particle tracking, when great care is expended, can provide excellent predictions; but this is easy only for fluids of invariant density.
- The r-equation method (donor-acceptor form) is to be recommended for flows with varying density.
- The two-fluid method can be expected to gain ground because of its flexibility and extensibility.
- A quite separate question is: how should the mass-continuity equation be formulated?

  $\frac{D \ln \rho}{Dt} + \mathrm{div}\,\vec{u} = 0$ is better than $\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho \vec{u}) = 0$

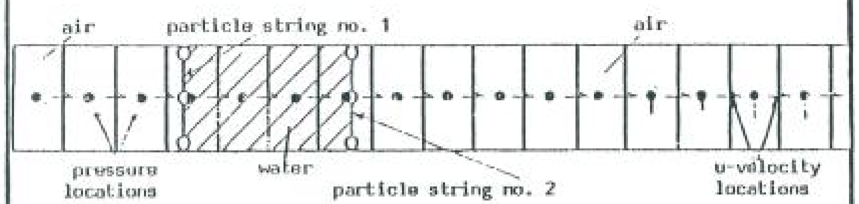
  The former is known as GALA (gas-and-liquid analyser).

PCH 1980	13/30	METHOD EMPLOYED IN EXAMPLES WHICH NOW FOLLOW.

- The determination of interface location: Particle-tracking.
- Continuity equation: GALA form.
- Work done by: T. T. Maxwell (liquid and gas) and
  A. G. Awn (gas and gas) at Imperial College.
- References: IC HTS Reports HTS/77/29 and HTS/79/8.
- Computational procedure employed for solving the equations of motion:
  SIMPLE (≡ semi-implicit procedure for pressure-linked equations).
- Dimensionality: all examples (but two) are 2D.
- Purpose of work: to demonstrate possibilities and establish numerical accuracy and physical realism.

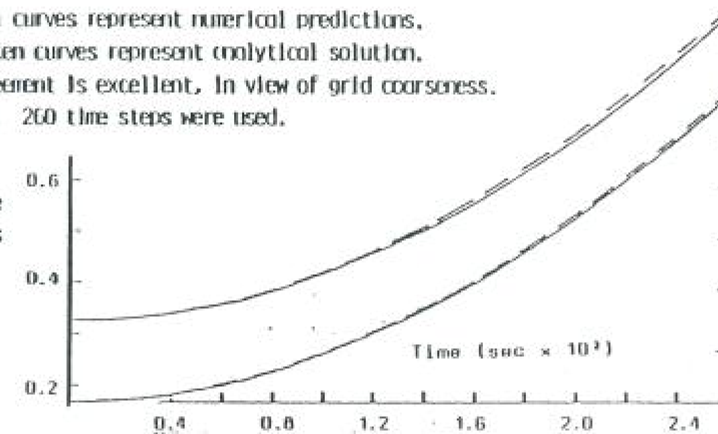
PCH 1980	14/30	APPLICATION 1 (T. T. MAXWELL). TEST OF NUMERICAL ACCURACY BY COMPARISON WITH ANALYTICAL SOLUTION.

- Problem considered: A plug of water in a pipe is subjected to a large pressure difference. Compute the subsequent motion.
- Method: As above, in one-dimensional version.
- Motive: Establish whether truncation error is acceptable.



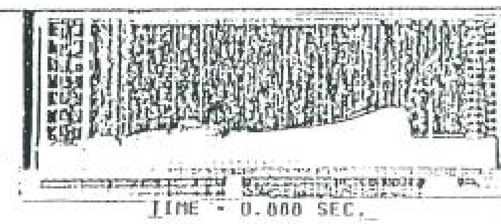
PCH 1980	15/30	APPLICATION 1. COMPARISON OF NUMERICAL WITH ANALYTICAL SOLUTION.

- Full curves represent numerical predictions.
- Broken curves represent analytical solution.
- Agreement is excellent, in view of grid coarseness.
  N.B. 260 time steps were used.

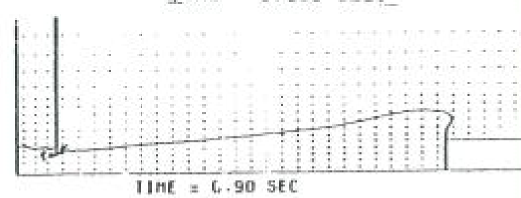


PCH 1980	16/30	APPLICATION 2. (T. T. MAXWELL) THE WAVE IN A TROUGH, "BREAKING" OVER AN OBSTACLE.

- Experiment: Water wave from left hits obstacle and "breaks" ⟶

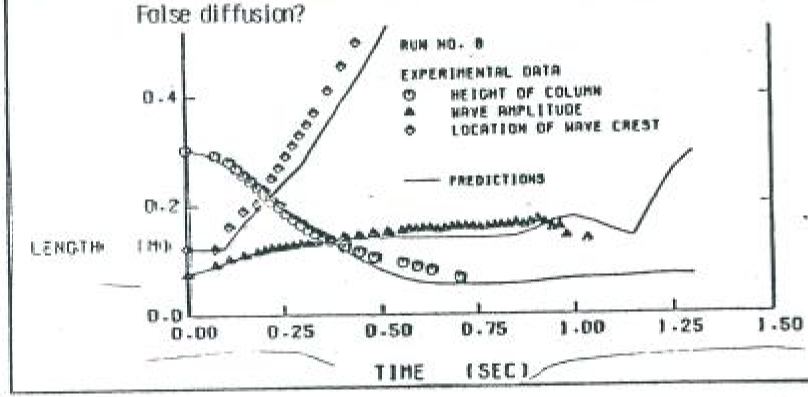


- Prediction: By above method, providing 2D transient flow fields in water and air ⟶
- Notes: The scales differ slightly; also the times.

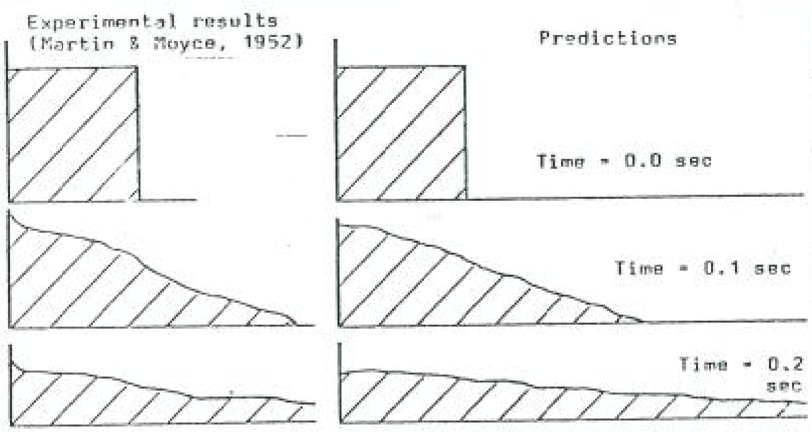


PCH 1980	17/30	APPLICATION 2, FURTHER COMPARISON WITH THE EXPERIMENT OF PANEL 16.

o Note: Predicted flow development is somewhat slower than experimental. False diffusion?



PCH 1980	18/30	APPLICATION 3 (T. T. MAXWELL). COLLAPSE OF A SEMI-CIRCULAR COLUMN OF LIQUID.

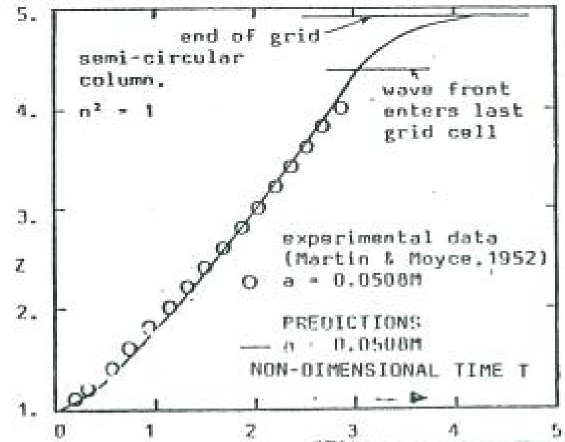


PCH 1980	19/30	APPLICATION 3. COMPARISON OF PREDICTED AND MEASURED WAVE LOCATIONS AT VARIOUS TIMES.

Details:

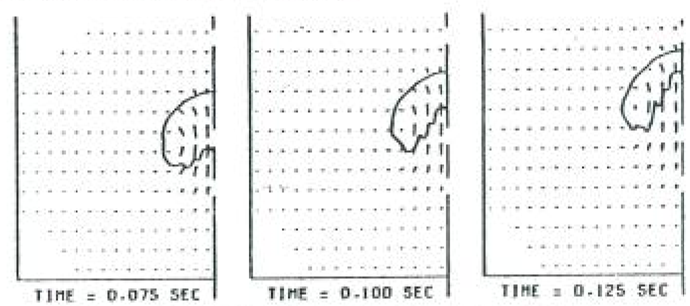
- 22 x 17 non-uniform grid;
- number of particles = 100 - 402;
- number of time steps = 60.

NON-DIMENSIONAL WAVE-FRONT LOCATION

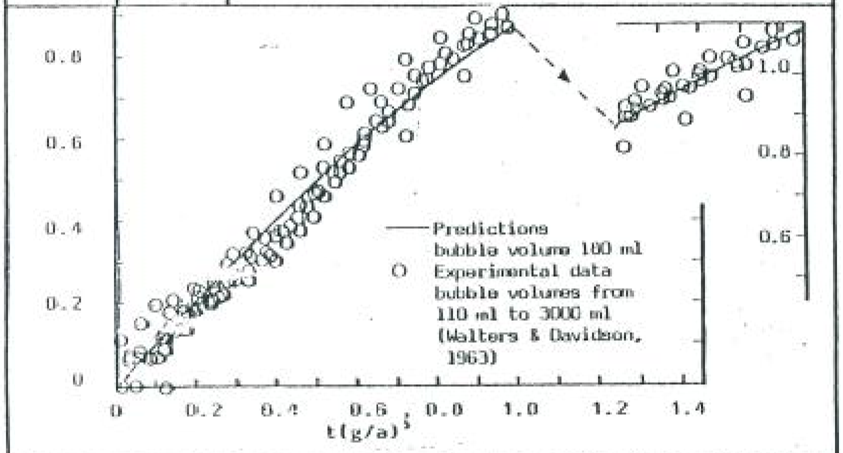


PCH 1980	20/30	APPLICATION 4. (T. T. MAXWELL) THE RISE OF A GAS BUBBLE THROUGH A LIQUID.

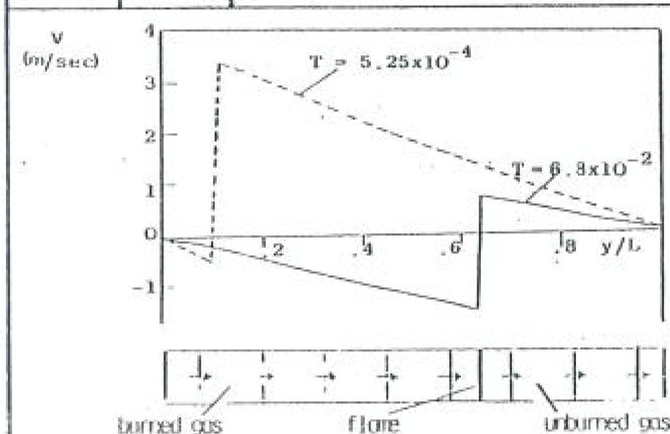
- Fluids: Air and water.
- Special features of calculations: Allowance is made for:-
  - compressibility of liquid;
  - distortion of water-pool surface.



PCH 1980	21/30	APPLICATION 4. COMPARISON OF PREDICTED AND MEASURED BUBBLE CENTROID TRAJECTORIES.

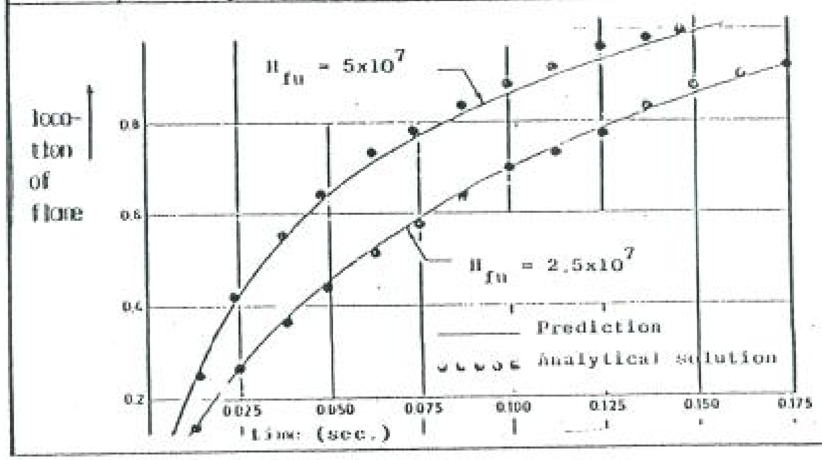


PCH 1980	22/30	APPLICATION 5. (A. G. AWN). 1D FLAME PROPAGATION IN A CLOSED TUBE, COMPUTED NUMERICALLY.

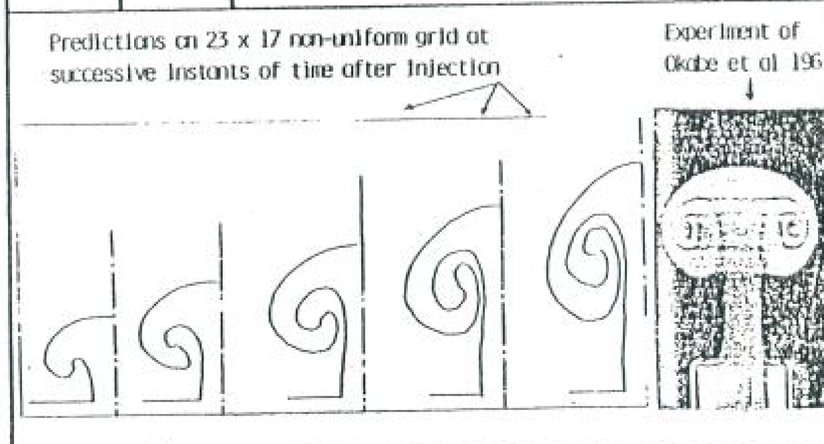


Notes:
(1) Gases are compressible.
(2) Flame speed is fixed relative to gas.

PCH 1980	23/30	APPLICATION 5. COMPARISON OF NUMERICAL WITH ANALYTICAL PREDICTION OF FLAME POSITIONS AT VARIOUS TIMES.

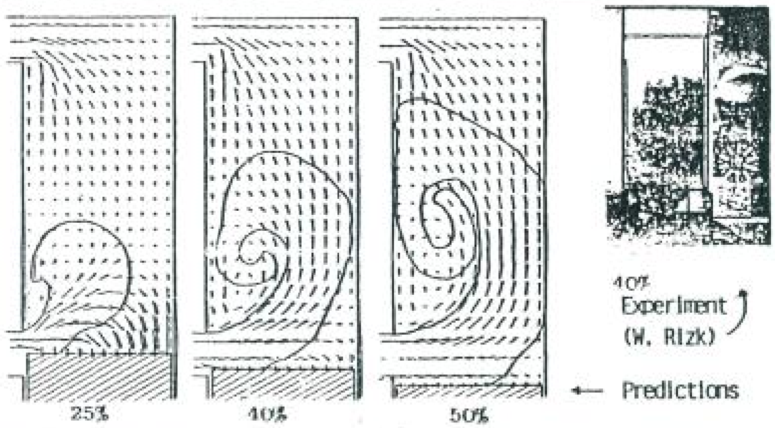


PCH 1980	24/30	APPLICATION 6. (A. G. Awn). INJECTION OF ONE LIQUID INTO ANOTHER.



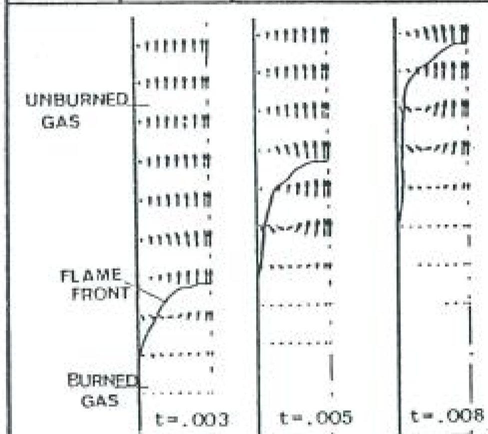
PCH 1980 | 25/30

APPLICATION 7. (A.G. Awn)
"SCAVENGING" IN A 2D MODEL OF A TWO-STROKE ENGINE.



PCH 1980 | 26/30

APPLICATION 8. (A.G. AWN)
FLAME PROPAGATION IN A TUBE.

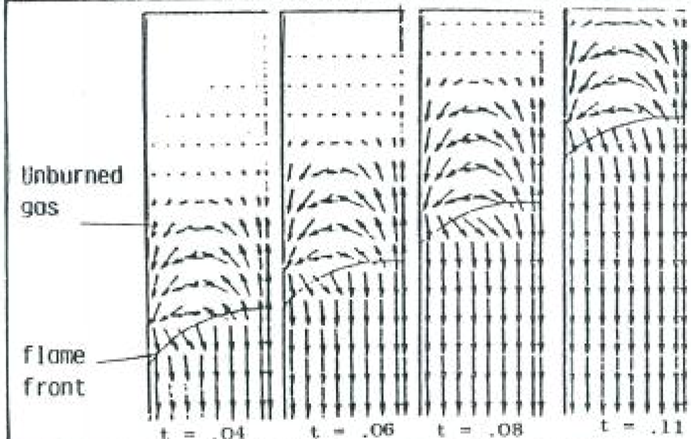


Notes:

- The flame is represented by an interface which advances relative to the gas at a prescribed speed.
- The base of the tube, where ignition occurs, is closed.
- Gravity makes the interface unstable.

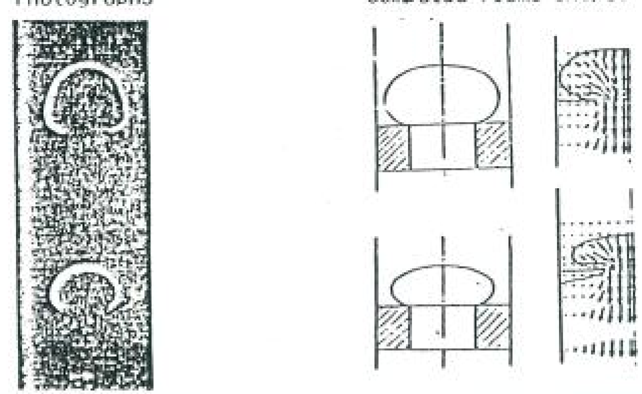
PCH 1980 | 27/30

APPLICATION 8
HERE THE UPPER END OF THE TUBE IS CLOSED



PCH 1980 | 28/30

APPLICATION 8.
PASSAGE OF FLAME THROUGH A CONSTRICTION IN A TUBE



NRTH 1980	1/28	Mathematical Methods in Nuclear-Reactor Thermal-Hydraulics. By D. Brian Spalding

- Mathematics
  - Formulation of finite-domain equations
  - Solution Procedures
- Physics
  - Inter-phase friction
  - Inter-phase heat and mass transfer
- Research Problems
  - Policy
  - Computer-code availability
  - The prospects for progress

NRTH 1980	2/28	The fundamental problems 1: Mathematical

In nuclear reactors, the flows which must be quantitatively predicted involve:-

- two phases, interspersed within and travelling relative to each other;
- heat, mass and momentum transfer between those phases;
- influences of gravity, pressure, inertia, viscous action, turbulence, surface tension.

The equations to be solved are therefore:

- numerous (6 velocities, 2 volume fractions, 2 enthalpies, 2 volume fractions, 1 or 2 pressures, etc.);
- coupled;
- non-linear But they must be soluble; for the phenomena exist.

NRTH 1980	3/28	The fundamental problems, 2: Physical

- The differential equations are coupled by auxiliary relations expressing:
  the rates of interchange, between the phases, of
  - heat, • mass, and • momentum,

  as functions of:
  - volume fractions, • degree and manner of sub-division,
  - relative velocity, • densities, • viscosities,
  - turbulence levels, • surface tensions, • etc.
- They also contain turbulence transport, generation and dissipation expressions.
- Knowledge of what expressions are best is completely absent.

NRTH 1980	4/28	The fundamental problems, 3: Policy

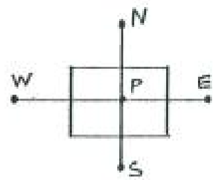
- Predictions must be made now; but the knowledge on which to base them is poor.
- Should the predictive codes therefore be shelved until the basic knowledge has been gathered?
- Should they be used now, with best-estimate input data?
- Should they be used only for upper- and lower-limit studies?
- How can the codes be used to further the research and so accelerate the gathering of knowledge?

NRTH 1980	5/28	How to solve the equations, 1: Features in common with single-phase flows.

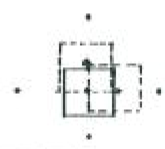
- The conservation equations are best expressed in terms of an array of control volumes, collectively covering the domain.
- The equations then become algebraic relations between flow variables (ϕ's), at adjacent grid points, of the form:

  $\phi_P = L(\phi_N, \phi_S, \phi_E, \phi_W)$,

  where $L() \equiv$ a linear function; but, strictly speaking, the coefficients depend on ϕ's .

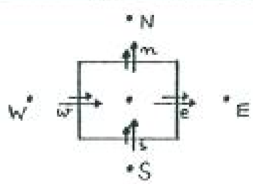


- Control volumes for velocities are best displaced relative to those of other variables.
- Pressures must be guessed, then corrected to preserve continuity.



NRTH 1980	6/28	How to solve the equations, 2: The greater interconnectedness of two-phase flow.

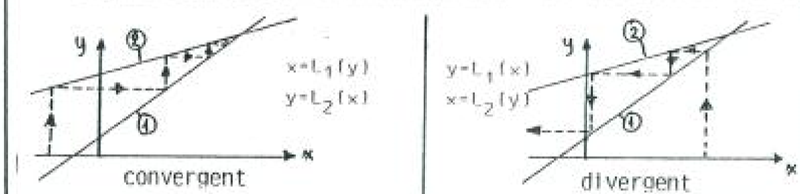
- Momentum balances:
  The pressure at P influences
  two velocities at n,
  two velocities at s,
  two velocities at e, and
  two velocities at w.



- Mass balances:
  The volume fractions at P are influenced by all these velocities.
- Volume compatibility:
  The two volume fractions must sum to unity.
- There are therefore 11 simultaneous equations to solve, even in the absence of heat/mass-transfer interactions, just for one cell, in 2D flow; and 15 in 3D flow.
- There may be 10,000 cells in the integration domain.

NRTH 1980	7/28	How to solve the equations, 3. A remark about "convergence".

- Non-linearity and coupling necessitate use of "guess-and-correct" procedures.
- Convergent procedures are hard to invent; and inventors of non-convergent procedures sometimes doubt the very existence of the solutions.
- This may explain the popularity of the (actually ill-founded) belief that the equations present an "ill-posed problem".
- The following example shows the crucial role of the adjustment order in determining whether the solution is found.

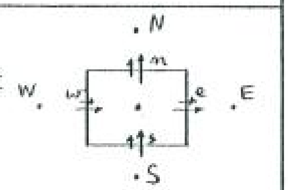


NRTH 1980	8/28	How to solve the equations,4. How we know they are soluble.

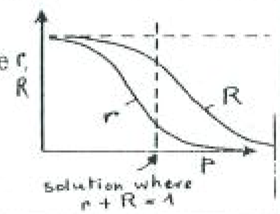
- If the pressure were fixed, the equations for the velocities and volume fractions would be de-coupled, and solubility would be certain.
- Therefore solubility depends on the possibility of finding the pressure field.
- No general theory exists for large coupled non-linear equation systems; but the properties of the equations for a single cell are usually shared by the multi-cell system.
- The question is therefore: can we prove that a unique value of pressure always exists, satisfying the single-cell equations?

NRTH 1980	9/28	How to solve the equations, 5. The single-cell-pressure analysis.

- Variables: r, R, p at P; velocities at n, s, e, w.
- Momentum relations: These dictate that outflow of each phase increases, and/or inflow decreases, as p increases.
- Individual phase mass balances: These dictate that volume fractions (r,R) decrease, as outflow increases and/or inflow decreases.



- Consequences:
  (1) increasing P decreases both volume fractions;
  (2) there always exists a unique pressure for which:
  $r + R = 1$
  Q.E.D.



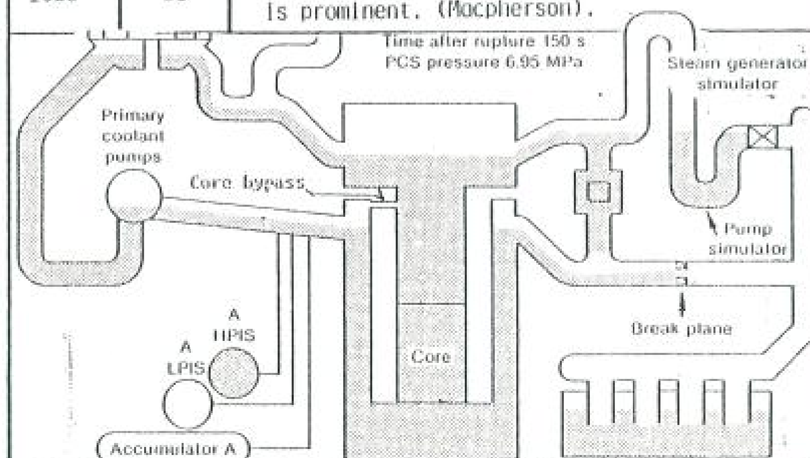
NRTH 1980	10/28	How to solve the equations, 6. Discussion.

- The conclusion holds for 1D, 2D, 3D, steady or unsteady situations.
- It is independent of the magnitudes of individual terms, cell dimensions, time step, etc.
- The conclusion is in accordance with physical intuition, and commonsense.
- It is considerations of this kind that guide the designers of successful solution algorithms, not the question of whether or not characteristics are imaginary.
- The solubility of the equations has been confirmed in practice by computations, during the last five years, by the Los Alamos group, by the author and colleagues, and by an increasing number of others.

NRTH 1980	11/28	How to solve the equations, 7. Examples of decisions to be made.

- The equation for r: $r = (r_i g_i + \dot{m})/g_o$,
  where $i \equiv$ inflow, $o \equiv$ outflow, $g \equiv$ density x velocity x area or density x volume ÷ time.
- Note: Until the converged solution has been obtained, r_i, g_i, $\dot{m}$ and g_o are only approximations.
- Questions: In order that r, evaluated from the equation, should be a better approximation;
  1. should r_i, g_i, g_o be given their values before or after application of pressure corrections?
  2. should changes in r be reduced or enlarged by a relaxation factor? (How calculated?)
  3. should g_o be replaced by some equivalent (in the converged state) function of g_i, G_i, G_o, R_i, R_o, etc.?
- Answers: 1. Before, 2. Yes, 3. Yes.

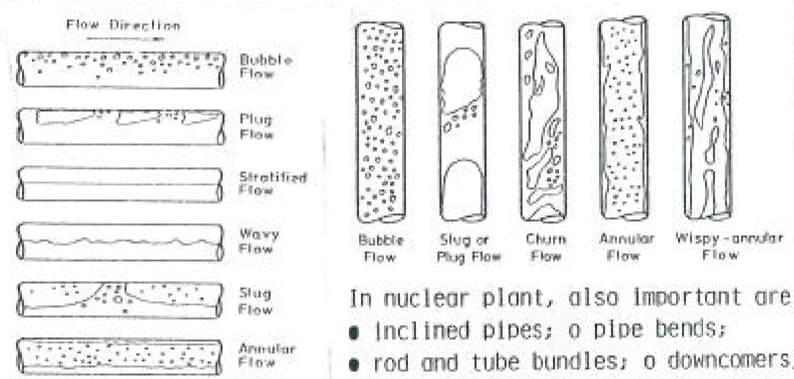
NRTH 1980	12/28	A reminder about the thermal-hydraulics of nuclear plant: gravity stratification is prominent. (Macpherson).



NRTH 1980	13 / 28	Nuclear-reactor thermal hydraulics; some phenomena of importance.

- Loss-of-coolant accident: A break in a vessel or pipe allows primary water (and steam) to escape (How rapidly? With what effects on pressures and temperatures elsewhere?)
- Refill: Emergency cooling water is pumped into ducts and/or vessels, and must flow to the core (what pressures and rates should be supplied? Where will the water go? How rapidly? How much steam will be formed? Where will it go?
- Reflood: The reactor core must be "flooded" with water. What temperature history will the fuel cladding sustain? How rapidly will the "quench front" advance?

NRTH 1980	14 / 28	Nuclear-reactor thermal hydraulics: some of the "flow regimes" observed in (more-or-less) steady pipe flow (Hewitt).



In nuclear plant, also important are:

- inclined pipes; o pipe bends;
- rod and tube bundles; o downcomers;
- baffles; etc.

NRTH 1980	15 / 28	Physical problems; Interphase friction, 1.

- The major question: How do f_x, f_y, f_z (≡ forces per unit volume exerted by second phase on first) depend on:
  - volume fractions of the phases: r, R?
  - velocity components of the phases: $u_x, U_x, u_y, U_y, u_z, U_z$?
  - length scales of sub-division: ℓ, L?
  - the densities, viscosities, surface tensions: d, D, μ, M, σ, S?
  - fluctuations of u's, r's, ℓ's, etc.?
- Notes: 1. Of the above variables, all have easily definable meanings except ℓ and L.
  2. ℓ ≡ L ≡ volume of mixture ÷ interfacial area is a useful definition; bubble or droplet size can be used.
  3. In truth, no single quantity can adequately characterize the interspersion of the phases.

NRTH 1980	16 / 28	Physical problems: Interphase friction, 2.

- A suitable (?) formula:

$$f_x = \bar{r}\,\bar{d}\,\bar{v}\,(U_x - u_x)\ell^{-1} \cdot F(\bar{d}\,\bar{v}\,\ell/\bar{\mu}, \bar{\sigma}\,\ell/d\bar{v}^2, \ldots)$$

where: $\bar{r} = [rR]^{n_r}$ [r, R ≡ volume fractions]

$\bar{d} = (d^{n_d} + D^{n_d})^{1/n_d}$ [d, D ≡ densities]

$\bar{v} = [(U_x - u_x)^2 + (U_y - u_y)^2 + (U_z - u_z)^2]^{\frac{1}{2}}$, etc.

and n_r, n_d, etc. are empirical constants.

- Comments: (1) The formula is qualitatively correct (e.g. $f_x \to 0$ as r, R, $U_x - u_x \to 0$).
  (2) Values of n's, and the form of F, can be deduced from:
  (a) experimental data;
  (b) mathematical analysis of idealisations.

NRTH 1980	17/28	Physical problems: Interphase friction, 3.

- Relevant experimental data: an f_x function can be deduced from (for example) "drift-flux" formulae, which give:

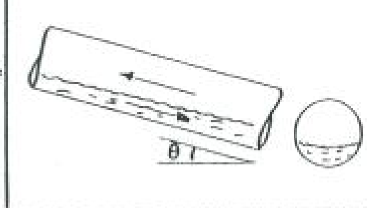
  $\tilde{v} = \tilde{v}\,[r, d, D, \sigma, \ldots\ldots\ldots]$,
- Method of deduction:
  - Reduce momentum equation as follows:

    convection (→0) + time-dependence (→0) + interphase friction = gravity + pressure gradient + wall friction.
  - Insert $\tilde{v}_r$ from drift-flux formula.
- Remark: Transformation of drift-flux data into f_x form permits their use in two-fluid computations.

NRTH 1980	18/28	Physical problems, Interphase friction, 4.

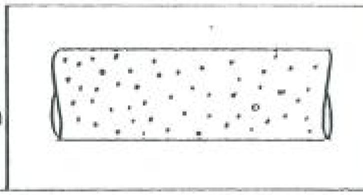
- An idealised model: stratified flow.
- Presume: • Interface roughness (waviness) increases

  with $f((U-u)^2/(g \cos\theta)).\ f(d/D))$
  - Time-average interface shape is plane.
  - Interface drag obeys rough-wall formulae.
- Results: An analytical expression can be derived connecting f_x with:
  - r ($f_x \to 0$ as $r \to 0$ or $r \to 1$),
  - u, U (f_x increases with $|U-u|$),
  - $g \cos\theta$,
  - μ, M,
  - d, D, etc.



NRTH 1980	19/28	Physical problems. Interphase friction, 5.

- A further idealised model: suspended-droplet flow.
- Presume: • Droplet size (L) is known, and uniform.
  - Droplets remain close to spherical in form.
  - Volume fraction of liquid (R) << 1.
- Results: An analytical expression can be derived connecting F_x with:
  - R ($F_x \to$ as $R \to$),
  - $u-U$,
  - d,
  - μ,
- Remark: Formula is reliable when presumptions are valid. But what determines L?



NRTH 1980	20/28	Physical problems, Interphase friction, 6.

- Other idealisable flow situations:
  - Slug (large bubble) flow.
  - Bubbly (small, uniform bubble) flow.
  - Annular flow.
- The main needs:
  - Criteria for transition from one flow regime to another, i.e. rules regarding when to switch formulae.
  - Differential equations connecting (among other things) the rate of change of the interspersion scales (ℓ, L) with relative velocity, surface tension, etc.

NRTH 1980	21/28	Physical problems: Heat and Mass Transfer.

- The need: Formulae are required connecting
  - heat fluxes from phase to interface with temperature differences;
  - mass fluxes from phase to interface with concentration differences;
  - and both with: r, R, ℓ, L, d, D, etc.
- The possibility: As for momentum transfer,
  - some information can be deduced from existing experimental information;
  - idealised models to lead to useful formulae;
  - the main uncertainty concerns the interspersion scale.

NRTH 1980	22/28	Physical problems Turbulent transfer

- The problem: If r, R, u_x, U_x, u_y, U_y, u_z, U_z are interpreted as time averages, what transfer of material occurs in addition to ru_x, RU_x, etc as a consequence of fluctuations?
  i.e., if primes denote fluctuating components, what are the magnitudes of $\overline{r'u_x'}$, $\overline{R'U_x'}$, etc?
- Current knowledge: This is mainly negative,
  e.g. $\overline{r'u_x'}$ is not proportional to $-\frac{\partial r}{\partial x}$.
- The needs: • Formulation of hypotheses, based on physical insight, and quantitative observation.
  - Systematic testing and refinement of hypotheses.
- Remark: The research task is a very large one; but worthwhile

NRTH 1980	23/28	Research-policy problems, 1. A reminder about turbulence models.

- In the 1950's it was recognised that turbulence was a complex phenomenon, suitable mainly for generating papers in applied mathematics.
- A few isolated geniuses (Kolmogorov, Prandtl), proposed a mathematical-modelling approach; but the equations were too difficult to solve; and the authorities were scornful.
- In the late 1960's however:
  1. the Kolmogorov-Prandtl proposals were uncovered or re-invented
  2. easy-to-use computer programs became available which permitted equations to be cheaply and frequently solved.
  3. graduate students were found who were willing to compare predictions with experiments, optimise constants, etc.
- Now, the applied mathematicians have moved in again; but meanwhile the engineer has gained something that he can use.

NRTH 1980	24/28	Research-policy problems, 2. Relevance of turbulence model to two-phase-flow research.

- Two-phase phenomena, like turbulence, are forbiddingly complex.
- Nevertheless, the framework of a mathematical model is in existence.
- A large body of empirical information is also available against which the implications of the models can be tested.
- What therefore are now needed are:
  1. the easy-to-use, and cheap, computer programs;
  2. the willing graduate students;
  3. support and encouragement from the community.

  Need 1 can now be met; and needs 2 and 3 when the importance and attainability of the goal are clearly seen.

NRTH 1980	25/28	Research activities. Concept formation.

- Computer models easily solve:

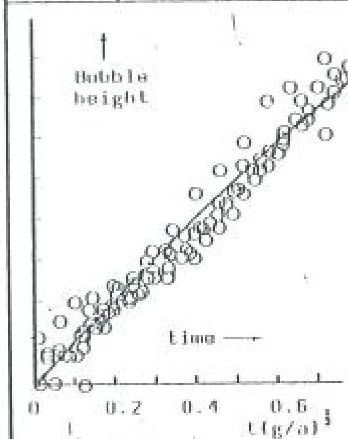
  $$\rho \frac{D\phi}{Dt} = \mathrm{div}\,(\Gamma\ \mathrm{grad}\ \phi) + \text{source of } \phi.$$

- Two vital questions are therefore: (1) What set of ϕ's can we best use to characterise two-phase flows? (ℓ? L? some measure of phase "connectedness"?)
  (2) What expressions should be employed to characterise their sources (and sinks).
- Analogy with turbulence modelling: Prandtl chose turbulence energy and length scale; Kolmogorov chose energy and frequency. Postulated sources were in part calculated, and in part guessed. Other choices are possible.
- Answers: None are yet available; even the questions are only now being recognised.

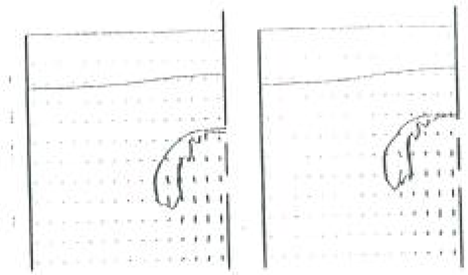
NRTH 1980	26/28	Research activities: Component-process studies.

- In one respect, two-phase-flow phenomena are easier to study than turbulence; for surface tension prevents the "significant scale" from becoming steadily smaller.
- This implies that the "source terms" in the ϕ equation can be deduced from detailed study of such processes as:
  - droplet collision and coalescence;
  - film disruption;
  - bubble rise and break-up.
- Computer modelling can play a part here also.
- What is needed is a good two-separated-phases hydrodynamics algorithm.

NRTH 1980	27/28	Research activities: Numerical computation of bubble motion (Maxwell).



Computations and measurements are in good agreement; many more such studies could (should?) be made.



NRTH 1980	28/28	Concluding remarks

- The mathematical problem of solving the two- (or multi-) fluid momentum, continuity and energy equations has been solved.
- Wide-spread use of inexpensive two-fluid computer codes, coupled with systematic measurements, could provide the needed knowledge about interphase friction, etc.
- New concepts are needed, especially those leading to computation of the degree of interspersion.
- Computer modelling can contribute significantly to the provision of quantitative source-and-sink relations for two-phase-flow equations.
- If the world's two-phase-flow researches were better co-ordinated, a single decade would bring sufficient knowledge for current needs.

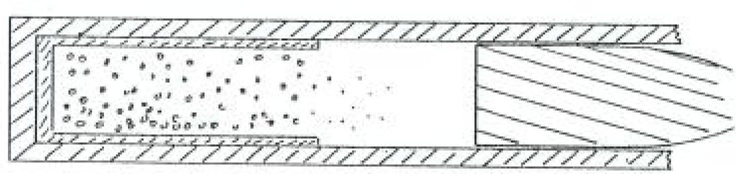
IPSA 1981 | 1/37

THE IPSA METHOD FOR COMPUTING MULTIPHASE FLOWS WITH INTERPHASE SLIP
by D BRIAN SPALDING

CONTENTS

IPSA 1981 | 2/37

PART 1. FLOW & COMBUSTION IN GUN BARRELS: THE PHYSICAL PHENOMENA



- Propellant particles burn at rates dependent upon pressure and temperature.
- Burning causes pressure and temperature increases.
- Pressure rises cause projectile and gas motion.
- Particles follow, dragged by gas.
- Casing burns. Wall is heated and eroded.

IPSA 1981 | 3/37

MATHEMATICAL PROBLEM OF PREDICTING GUN-BARREL FLOWS: INDEPENDENT VARIABLES

- Time: process is essentially unsteady.
- Axial Distance: ignition occurs at one end, travels axially. Large variations of temperature, pressure, velocity exist.
- Radial Distance: heat transfer and wall friction cause radial variations; the casing burns; radial gradation of propellant properties may be built in.
- Circum-ferential location: propellants often start 'off-centre'. Erosion causes ovality.
- Conclusion: at least 3, possibly 4 independent variables are involved.

IPSA 1981 | 4/37

MATHEMATICAL PROBLEM.........: DEPENDENT VARIABLES

- Gas: temperature; pressure; axial, radial (and circumferential) velocities; 2 turbulence properties; volume fraction; composition.
- Particles: temperature; (pressure); axial, radial (and circumferential) velocities; particle size; volume fraction; composition.
- Walls: temperature.
- Casing: as for particles.
- Radiation: fluxes in axial, radial (and circumferential) directions.
- Conclusion: 2 pressures, 4 (or 6) velocities, 2 (or more) concentrations, 2 temperatures, 2 turbulence quantities, 3 volume fractions, 2 (or 3) radiation fluxes, will suffice.

IPSA 1981 | 5/37 | MATHEMATICAL PROBLEM.........: DIFFERENTIAL EQUATIONS

- All 17 (or more) dependent variables (ϕ's) obey:

$$\frac{\partial}{\partial t}(r\rho\phi) - \mathrm{div}(r(\rho\vec{v}\phi - \Gamma_\phi \mathrm{grad}\,\phi)) = rS_\phi$$

where: $r, \rho, \vec{v}, \Gamma_\phi, S_\phi$ are, for phase in question, volume fraction, density, velocity vector, exchange coefficient, source rate per unit volume.

- S_ϕ represents:
  - pressure gradient and interphase friction for velocities,
  - interphase mass transport for volume fractions,
  - interphase heat transfer and radiative absorption for enthalpies,
  - chemical - transformation rates for concentration,
  - etc.

IPSA 1981 | 6/37 | MATHEMATICAL PROBLEM.........: AUXILIARY ALGEBRAIC RELATIONS

- Volume Compatibility: r's for phases sum to 1.
- Thermodynamics: p, ρ, T, concentrations are connected algebraically.
- Chemical Kinetics: S_ϕ (p, ρ, T, concentration).
- Turbulence: generation, dissipation, transport expressions for energy, etc.
- Interphase friction: how particle ~ gas momentum exchange depends on diameter, etc.
- Interphase heat transfer: how it depends on diameter, transport properties, etc.
- Interphase mass transfer: how the burning rate depends on pressure, particle temperature, relative velocity of gas, etc.

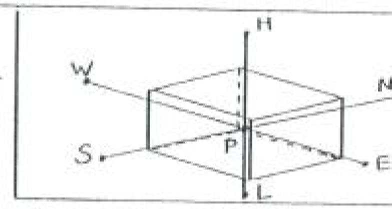
IPSA 1981 | 7/37 | MATHEMATICAL PROBLEM.........: DISCUSSION

- Initial conditions must be prescribed for all ϕ's.
- Boundary conditions may be in terms of ϕ's, ϕ-fluxes, or $\phi \sim \phi$-flux relations.
- The integration domain may extend into the barrel and the projectile.
- The differential equations are inherently non-linear.
- They are multiply interconnected.
- They are parabolic in form.
- The magnitude of the problem is large, eg in comparison with those of classical aerodynamics.
- Nevertheless, it is soluble.

Note: 'Proofs' have been published that the problem is 'ill-posed'. These proofs have been proved to be erroneous.

IPSA 1981 | 8/37 | SOLUTION PROCEDURE: FINITE-DOMAIN EQUATIONS (FDE's)

- Differential equations, integrated (with interpolation assumptions) over finite sub-domains (cells), yield FDE's.
- These have form:



$$a_P\phi_P = a_N\phi_N + a_S\phi_S + a_E\phi_E + a_W\phi_W + a_H\phi_H + a_L\phi_L + b$$

- a's and b's are also functions of the ϕ's.
- The fully-implicit form is preferable, i.e.
  - all ϕ's are end-of-time-interval values
  - beginning-of-interval value appears in b.
- Solution techniques for simultaneous linear equations are used, iteratively.

IPSA 1981	9/37	SOLUTION PROCEDURE: IPSA ALGORITHM: THE MAIN IDEA

- A difficulty:
  The six velocity-component and two continuity equations are strongly linked; and the shared pressure is (pressures are?) unknown. How to achieve simultaneous solution?
- The IPSA technique:
  (1) Guess pressure (s).
  (2) Solve volume-fraction equations.
  (3) Solve momentum equations.
  (4) Compute errors in phase-conservation equation.
  (5) Compute combined (weighted) errors.
  (6) Compute pressure corrections which eliminate these.
- Details will be given below.

IPSA 1981	10/37	COMPUTER PROGRAMS FOR TWO-PHASE FLOW 1. GENMIX 2P

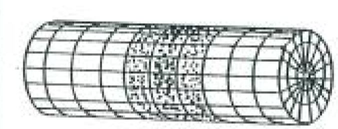
- GENMIX 2P is a two-phase version of GENMIX (Spalding 1977).
- It handles 2D parabolic flows, either 2D steady or 1D unsteady, for either 1 or 2 phases.
- Its coordinates are:
  (1) longitudinal distance, or time.
  (2) Non-dimensional stream function, $\int \rho u dy$, or $\int \rho dy$ of the <u>first</u> (continuous) phase.
- GENMIX 2P is convenient for gun-barrel problems because the $\int \rho dy$ coordinate remains fixed in extent as the projectile moves.
- GENMIX 2P is a useful test vehicle for turbulence models, interphase friction laws, etc.

IPSA 1981	11/37	COMPUTER CODES FOR TWO-PHASE FLOW 2. PHOENICS - TIBALT

- PHOENICS is a computer-code system, comprising a central equation solver, 'EARTH', and a set of satellites.
- TIBALT is a satellite for Three-dimensional Internal Ballistics.
- Other satellites using EARTH's two-phase capability model steam generators and nuclear reactors (steam + water), diesel engines (oil droplets + air), and coal combustors (coal particles + air).
- EARTH embodies the IPSA algorithm and solves up to 25 differential equations in 1, 2 or 3 dimensions.
- Special burning-rate, interphase friction and turbulent-transport laws can be supplied as 'Ground Stations'.

IPSA 1981	12/37	COMPUTER CODES FOR TWO-PHASE FLOW 3. SOME DETAILS OF EARTH

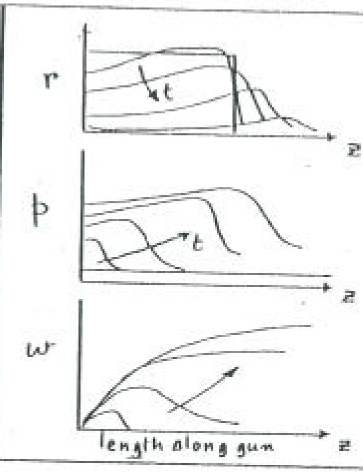
- Grid:
  Cartesian or polar; fixed or expanding/contracting.
- Storage:
  Only 3 'slabs' of material need be in core at one time; use of disc storage for remainder means that very fine grids are usable.



- Computer time:
  Fully-implicit formulation of finite-domain equations permits large time steps; use of fast solvers renders computer times modest.
- Maintenance:
  Since EARTH is the same for all satellites, it is in constant use, and is continuously maintained.

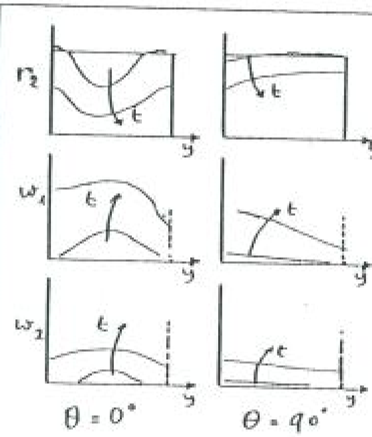
IPSA 1981	13/37	RESULTS FROM TIBALT, 1D. (Sketches, representing qualitative behaviour shown by actual calculations)

- Problem:
  Propellant charge is ignited at $z = 0$;
  ignition wave propagates;
  pressure rises,
  projectile moves.
- Dependent variables:
  Velocities of gas and particles.
  Temperatures of ditto.
  Volume fractions of ditto.
  Pressures of ditto.
- Processes considered:
  Interphase mass, heat and momentum transport.



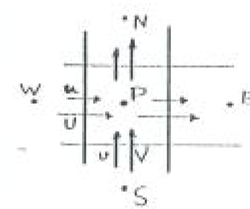
IPSA 1981	14/37	RESULTS FROM TIBALT, 3D

- Problem:
  As for 1D, but ignition point is at $y = 1/2 y_{max}$.
- Further processes considered:
  None; but off-centre ignition causes circumferential non-uniformities.
- Results: as sketched.
- Computer Times:
  (UNIVAC 1108 equivalent)
  $= 10^{-4}$ s per cell,
  equation and time step.



IPSA 1981	15/37	PART 2. THE IPSA PROCEDURE DESCRIPTION

- To find:
  volume fractions, r, R;
  velocities, u, U, v, V;
  pressures, p, P;
  enthalpies: h, H.



- Procedure
  (case of negligible P):
  (1) Guess u, U, v, V.
  (2) Solve for r, R (satisfying $r + R = 1$).
  (3) Guess p.
  (4) Solve for u, U, v, V.
  (5) Compute e, E ($\equiv$ errors in continuity equations).
  (6) Compute pressure correction, p′, zeroing e/w + E/W.
  (7) Adjust p, u, U, v, V accordingly. (8) Repeat.

IPSA 1981	16/37	THE IPSA PROCEDURE: THE r-EQUATION

- Definition: $g \equiv$ density × velocity × area or density × volume/δt, subscript $i \equiv$ inflow, $o \equiv$ outflow.
- Conservation equation (based on "upwind differences"):
  $$r = \Sigma(rg)_i / \Sigma g_o$$
- Comment: Although correct, when g_i and g_o are correct, this permits divergence when g's are only approximate.
- Recommendation: Add to denominator the (nominally zero) quantity: $(\Sigma(rg)_i/\Sigma g_o - r) + (\Sigma(RG)_i/\Sigma G_o - R)$.
- Resulting formula (since $r + R = 1$):
  $$r = \frac{\Sigma G_o \cdot \Sigma(rg)_i}{\Sigma G_o \cdot \Sigma(rg)_i + \Sigma g_o \Sigma(RG)_i}$$

IPSA 1981	17/37	THE IPSA PROCEDURE: 'WEIGHTING' OF CONTINUITY ERRORS

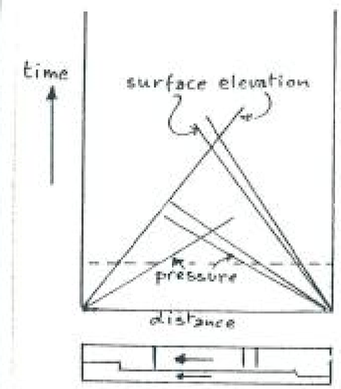
- Use: $\partial u/\partial p$, $\partial U/\partial p$, $\partial v/\partial p$, $\partial V/\partial p$ are known.
  Hence, since $e + E$ depends on u, v & U, V,
  $\partial e/\partial p$ & $\partial E/\partial p$ can be deduced (r, R fixed).
  Hence, p' can be deduced from:
  $$p'(\frac{\partial e}{\partial p}/w + \frac{\partial E}{\partial p}/W) = -(e/w + E/W).$$
- Recommendation: Let w & W be densities of the two phases.
- Result: p' is chosen to satisfy "volumetric-continuity" equation.
- Remarks: This is a good general rule; but one can do better in some circumstances.

IPSA 1981	18/37	THE IPSA PROCEDURE: DISCUSSION

- IPSA has been successfully used for:
  - steam ~ water flows
  - pulverised-coal combustion
  - burning of gun propellants
  - deposition of dust from gaseous streams
- Often P (the 'second pressure') can not be neglected eg for 'stratified flows', 'particle packing'.
- IPSA works for 1.E-9 ≤ r ≤ 1.E-9; this is probably sufficient.
- Special versions handle: 'donor-acceptor' differencing; particle-size calculation; other special effects.

IPSA 1981	19/37	PART 3. TWO-PHASE WAVE PROPAGATION IN DUCTS. THE PROBLEM CONSIDERED

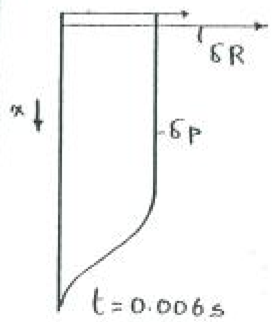
- Dimensionality: 1D
- Transience: Unsteady
- Variables: u, U, r, R, p, P.
- Auxiliary relations:
  f = const. r R (U - u)
  R $\partial P/\partial x$ = const. $\partial R^2/\partial x$.
- Initial conditions: both fluids have equal uniform velocities; uniform p.
- Boundary conditions: both duct ends are closed.



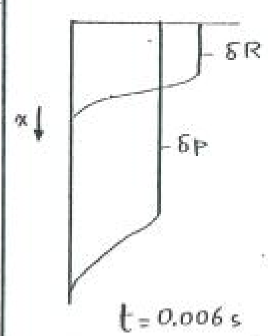
IPSA 1981	20/37	TWO-PHASE WAVE PROPAGATION: COMPUTATIONAL DETAILS

- Computer code: PHOENICS with PLANT satellite.
- Fluids: Compressible gas; incompressible liquid (ρ_ℓ = 1000 ρ_g).
- Grid: 100 intervals, velocity nodes staggered.
- Computer time: 79s (Perkin Elmer 3220) for .006s flow time.
- Duct length: 10m.
- Initial fluid velocities: - 4 m/s.
- Time step: 0.2×10^{-3} s.
- Interphase friction: = const x r x R.
- Second-pressure formulation: const x R^2.
- Boundary conditions: u = U = 0 at ends.
- Heat transfer & wall friction: zero.

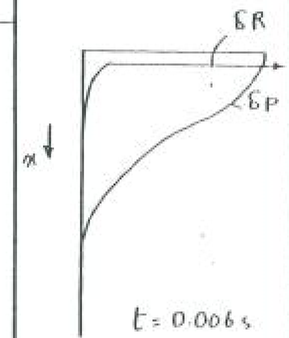
IPSA 1981	21/37	TWO-PHASE WAVE PROPAGATION, CASE 1: NO INTERPHASE FRICTION, NO 2ND PRESSURE		
IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	15	240	.04	-4.0
3	15	0	.04	-4.0
5	15	0	.04	-4.0
7	15	0	.04	-4.0
9	15	0	.04	-4.0
11	15	0	.04	-4.0
13	15	0	.03	-4.0
15	15	0	-.04	-4.0
17	14	0	-.29	-4.0
19	13	0	-.81	-4.0
21	10	0	-1.56	-4.0
23	7	0	-2.35	-4.0
25	4	0	-3.02	-4.0



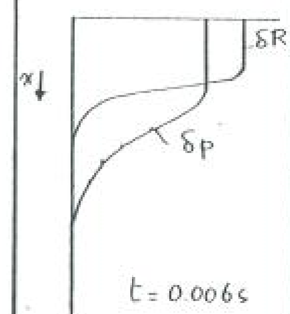
IPSA 1981	22/37	TWO-PHASE WAVE PROPAGATION, CASE 2: NO INTERPHASE FRICTION, SECOND PRESSURE ACTIVE		
IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	15	40	0	.01
3	15	40	0	.10
5	15	36	.02	-.98
7	15	11	.04	-3.29
9	15	1	.04	-3.93
11	15	0	.04	-4.00
13	15	0	.03	-4.00
15	15	0	-.04	-4.00
17	14	0	-.29	-4.00
19	13	0	-.81	-4.00
21	10	0	-1.56	-4.00
23	7	0	-2.35	-4.00
25	4	0	-3.02	-4.00



IPSA 1981	23/37	TWO-PHASE WAVE PROPAGATION, CASE 3: MODERATE INTERPHASE FRICTION, NO 2ND PRESSURE		
IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	37	107	-.12	-2.32
3	32	7	-.43	-2.69
5	27	5	-.80	-2.97
7	22	4	-1.20	-3.21
9	18	3	-1.62	-3.42
11	14	1	-2.04	-3.59
13	11	1	-2.43	-3.73
15	8	0	-2.80	-3.83
17	5	0	-3.19	-3.90
19	3	0	-3.49	-3.95
21	2	0	-3.71	-3.97
23	1	0	-3.85	-3.99
25	0	0	-3.93	-4.00

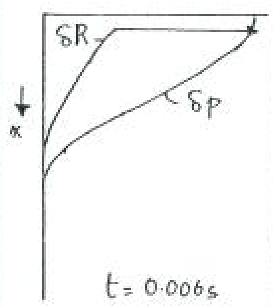


IPSA 1981	24/37	TWO-PHASE WAVE PROPAGATION, CASE 4: MODERATE INTERPHASE FRICTION, 2ND PRESSURE ACTIVE		
IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	26	36	-.06	.05
3	27	36	-.17	.20
5	27	33	-.28	-.59
7	26	13	-.63	-2.45
9	21	4	-1.14	-3.18
11	16	2	-1.69	-3.46
13	12	2	-2.21	-3.65
15	9	1	-2.68	-3.79
17	6	1	-3.09	-3.88
19	3	0	-3.43	-3.94
21	2	0	-3.68	-3.97
23	1	0	-3.84	-3.99
25	0	0	-3.93	-4.00



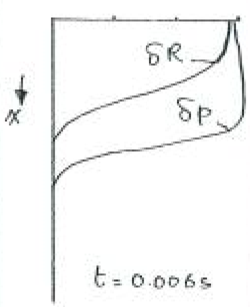
IPSA 1981	25/37	TWO-PHASE WAVE PROPAGATION, CASE 5: HIGH INTERPHASE FRICTION, NO 2ND PRESSURE

IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	49	105	- .05	- .42
3	44	24	- .35	-1.09
5	36	16	- .96	-1.91
7	25	10	-1.77	-2.09
9	15	5	-2.59	-3.29
11	8	2	-3.24	-3.67
13	4	1	-3.65	-3.87
15	1	0	-3.86	-3.95
17	0	0	-3.95	-3.98
19	0	0	-3.98	-4.00
21	0	0	-4.00	-4.00
23	0	0	-4.00	-4.00
25	0	0	-4.00	-4.00



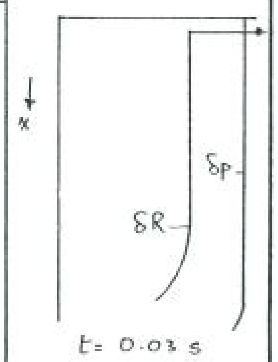
IPSA 1981	26/37	TWO-PHASE WAVE PROPAGATION, CASE 6: HIGH INTERPHASE FRICTION, 2ND PRESSURE ACTIVE

IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	33	32	- .03	.04
3	34	31	- .08	.12
5	36	29	- .12	- .26
7	32	18	- .70	-1.49
9	22	10	-1.73	-2.60
11	12	5	-2.72	-3.33
13	6	2	-3.40	-3.73
15	2	1	-3.76	-3.90
17	1	0	-3.92	-3.97
19	0	0	-3.98	-3.99
21	0	0	-3.99	-4.00
23	0	0	-4.00	-4.00
25	0	0	-4.00	-4.00



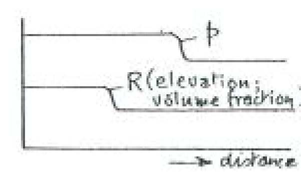
IPSA 1981	27/37	TWO-PHASE WAVE PROPAGATION, CASE 5 LATER TIME: HIGH INTERPHASE FRICTION, NO 2ND PRESSURE

IX	$\delta p \times 10^{-2}$	$\delta R \times 10^{3}$	u	U
1	51	114	0	0
3	51	36	0	0
5	51	36	0	0
7	51	36	0	0
9	51	36	0	0
11	51	35	0	- .01
13	51	35	- .01	- .03
15	51	35	- .03	- .06
17	51	34	- .05	- .12
19	51	33	- .10	- .21
21	50	32	- .19	- .34
23	50	30	- .31	- .53
25	49	28	- .50	- .77



IPSA 1981	28/37	TWO-PHASE WAVE PROPAGATION: CONCLUSIONS

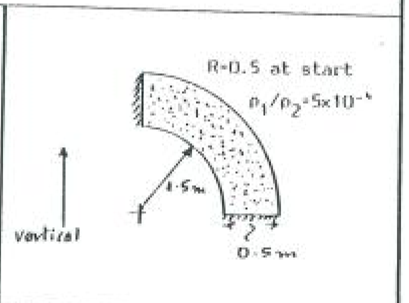
- IPSA works satisfactorily both with and without interphase friction and second pressure.
- With no second pressure, the wave thickness depends on interphase friction but the speed and pressure rise do not.



- With second pressure, there are two waves, with speeds and amplitudes depending on interphase friction.
- Questions about 'the' sound speed in a two-phase mixture are meaningful only if all circumstances are described.

IPSA 1981	29/37	PART 4. SEDIMENTATION IN A CURVED DUCT: THE PROBLEM

- A segment of a two-dimensional duct is considered (see sketch).
- The dense and light fluids are uniformly dispersed at the start.
- The duct ends are closed.
- There is interphase friction (= const x r x R per unit volume).



- The task is to predict the 'sedimentation' process.
- The liquid is expected to fall; but what flows will be generated as it does so?

IPSA 1981	30/37	THE CURVED-DUCT PROBLEM COMPUTATIONAL DETAILS

- Computer code: PHOENICS plus GUSSIE satellite.
- Grid: NX 10, NZ 1 (Polar)
- Fluids: Both liquid and gas, supposed incompressible.
- Computer: CDC 6400.
- Computer time: 10s for 1s flow time.
- Time step: 0.05s.
- Interphase-friction constant: Moderate.
- Initial conditions: Both fluids at rest.
- Special practices: None.
- Number of data entries to define the problem, the run conditions and the print out: 46.

IPSA 1981	31/37	THE VOLUME FRACTIONS AND PRESSURES AFTER 0.1 s

VALUES OF R2

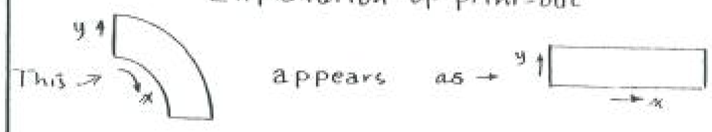
IY=10	1.159E-01	2.237E-01	3.647E-01	5.283E-01	7.161E-01
IY= 8	1.704E-01	3.364E-01	4.698E-01	5.412E-01	6.779E-01
IY= 6	2.021E-01	3.499E-01	4.469E-01	1.772E-01	5.155E-01
IY= 4	2.779E-01	3.041E-01	4.404E-01	4.024E-01	4.387E-01
IY= 2	1.564E-01	2.666E-01	2.932E-01	2.478E-01	4.966E-01
IX=	1	3	5	7	9

VALUES OF P

IY=10	-3.436E+03	-3.729E+03	-2.844E+03	-4.537E+02	3.284E+03
IY= 8	-3.171E+03	-3.250E+03	-2.202E+03	1.026E+02	3.626E+03
IY= 6	-2.813E+03	-2.654E+03	-1.528E+03	6.060E+02	3.349E+03
IY= 4	-2.320E+03	-1.978E+03	-8.578E+02	1.070E+03	3.536E+03
IY= 2	-1.368E+03	-1.526E+03	-4.390E+02	1.435E+03	3.871E+03
IX=	1	3	5	7	9

>.5 <.5 <0 >0 → x

Explanation of print-out



IPSA 1981	32/37	THE VELOCITIES AT 0.1 s U ≡ CIRCUMFERENTIAL V ≡ RADIAL 1 ≡ LIGHT PHASE 2 ≡ HEAVY PHASE

VALUES OF U1
CICS SUB-PROGRAM NO. E001, ISSUED BY CHAM UK 24.04.81

IY=10	-9.057E-01	-1.705E+00	-1.697E+00	-1.346E+00	-1.128E+00
IY= 8	-1.351E-01	-5.292E-01	-7.031E-01	-8.042E-01	-8.331E-01
IY= 6	1.077E-01	1.035E-01	-2.739E-02	-3.529E-01	-5.458E-01
IY= 4	2.351E-01	4.913E-01	4.448E-01	2.023E-01	-9.811E-02
IY= 2	2.575E-01	5.167E-01	3.560E-01	3.654E-01	7.882E-01
IX=	1	3	5	7	9

VALUES OF U2

IY=10	-9.124E-01	-1.644E+00	-1.525E+00	-1.164E+00	-8.820E-01
IY= 8	-2.226E-01	-4.721E-01	-3.669E-01	-6.372E-01	-6.201E-01
IY= 6	1.163E-01	1.737E-01	3.873E-02	-2.261E-01	-3.136E-01
IY= 4	2.598E-01	5.319E-01	5.709E-01	4.452E-01	1.936E-01
IY= 2	3.053E-01	6.100E-01	5.433E-01	5.927E-01	1.108E+00
IX=	1	3	5	7	9

VALUES OF V1

IY=10	0.	0.	0.	0.	0.
IY= 8	-3.875E-01	-1.079E-01	1.397E-01	1.046E-01	9.277E-02
IY= 6	-4.486E-01	-1.550E-01	7.366E-02	7.595E-02	1.123E-01
IY= 4	-3.409E-01	-1.374E-01	1.963E-02	-7.074E-03	1.200E-01
IY= 2	-1.313E-01	-3.562E-02	3.335E-02	-3.655E-02	6.318E-02
IX=	1	3	5	7	9

VALUES OF V2

IY=10	0.	0.	0.	0.	0.
IY= 8	-5.325E-01	-2.655E-01	-5.903E-03	6.235E-03	9.582E-02
IY= 6	-6.490E-01	-3.310E-01	-7.255E-02	-2.238E-02	1.243E-01
IY= 4	-5.427E-01	-3.220E-01	-1.328E-01	-1.030E-01	8.701E-02
IY= 2	-4.307E-01	-2.375E-01	-1.640E-01	-1.574E-01	1.421E-02
IX=	1	3	5	7	9

IPSA 1981	33/37	R's AND p's AT 0.2 s AND 0.3 s

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	2.103E-02	5.776E-02	4.161E-01	7.749E-01	8.785E-01
IY= 8	1.300E-01	1.457E-01	4.457E-01	7.658E-01	8.593E-01
IY= 6	1.319E-01	2.171E-01	3.870E-01	6.959E-01	8.050E-01
IY= 4	1.376E-01	2.023E-01	3.306E-01	5.507E-01	6.707E-01
IY= 2	1.462E-01	1.292E-01	2.246E-01	3.767E-01	5.864E-01

VALUES OF P

IY	IX=1	3	5	7	9
IY=10	-3.271E+03	-3.196E+03	-2.819E+03	-5.599E+02	4.283E+03
IY= 8	-3.124E+03	-2.993E+03	-2.162E+03	1.700E+02	4.405E+03
IY= 6	-2.861E+03	-2.658E+03	-1.541E+03	7.797E+02	4.501E+03
IY= 4	-2.536E+03	-2.236E+03	-9.835E+02	1.328E+03	4.806E+03
IY= 2	-2.204E+03	-1.923E+03	-8.423E+02	1.730E+03	5.034E+03

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	4.395E-03	1.807E-02	6.591E-02	8.567E-01	9.392E-01
IY= 8	1.061E-02	4.410E-02	1.416E-01	8.949E-01	9.099E-01
IY= 6	3.415E-02	6.658E-02	2.339E-01	8.257E-01	8.505E-01
IY= 4	1.088E-01	1.456E-01	3.972E-01	7.562E-01	7.578E-01
IY= 2	1.785E-01	2.550E-01	4.674E-01	6.771E-01	7.799E-01

VALUES OF P

IY	IX=1	3	5	7	9
IY=10	-2.919E+03	-2.914E+03	-2.734E+03	-1.153E+03	4.315E+03
IY= 8	-2.703E+03	-2.365E+03	-2.562E+03	-3.249E+02	4.490E+03
IY= 6	-2.833E+03	-2.751E+03	-2.254E+03	3.101E+02	4.697E+03
IY= 4	-2.730E+03	-2.541E+03	-1.735E+03	1.115E+03	5.101E+03
IY= 2	-2.458E+03	-2.170E+03	-1.064E+03	1.881E+03	5.427E+03

IPSA 1981	34/37	R's AT 0.4, 0.5, 0.6, 0.7 s

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	3.723E-04	1.341E-03	2.422E-02	9.174E-01	9.261E-01
IY= 8	3.122E-03	5.995E-03	5.932E-02	9.195E-01	9.213E-01
IY= 6	9.173E-03	1.553E-02	2.023E-01	8.917E-01	8.499E-01
IY= 4	2.101E-02	4.484E-02	3.320E-01	8.322E-01	7.613E-01
IY= 2	5.050E-02	2.476E-01	5.637E-01	7.112E-01	8.330E-01

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	5.883E-05	1.739E-04	7.346E-03	9.235E-01	9.564E-01
IY= 8	2.493E-04	8.184E-04	1.503E-02	9.141E-01	9.128E-01
IY= 6	7.434E-04	2.363E-03	5.038E-02	9.506E-01	8.254E-01
IY= 4	2.607E-03	6.293E-03	7.347E-01	8.420E-01	7.777E-01
IY= 2	1.136E-02	1.047E-01	7.578E-01	7.299E-01	8.402E-01

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	2.032E-04	8.243E-05	2.513E-02	9.368E-01	9.624E-01
IY= 8	4.554E-04	3.396E-04	1.425E-02	9.164E-01	9.210E-01
IY= 6	5.111E-04	3.431E-04	1.271E-02	8.655E-01	8.464E-01
IY= 4	5.526E-04	3.003E-04	3.675E-01	8.612E-01	8.467E-01
IY= 2	1.190E-03	1.191E-02	5.925E-01	9.053E-01	9.531E-01

VALUES OF R2

IY	IX=1	3	5	7	9
IY=10	1.816E-05	2.164E-05	1.898E-02	9.563E-01	9.316E-01
IY= 8	9.237E-05	1.422E-04	9.883E-03	9.492E-01	9.526E-01
IY= 6	1.673E-04	8.140E-05	6.189E-03	9.046E-01	9.013E-01
IY= 4	2.268E-04	2.072E-04	7.291E-03	8.956E-01	8.767E-01
IY= 2	3.160E-04	4.318E-04	9.875E-01	9.995E-01	1.000E+00

IPSA 1981	35/37	u's (CIRCUMFERENTIAL) v's (RADIAL) AT 0.7 s

VALUES OF U1

IY	IX=1	3	5	7	9
IY=10	-1.142E-01	7.833E-01	4.653E-01	-1.556E+00	-1.433E+00
IY= 8	-4.586E-02	7.404E-01	-7.133E-02	-9.802E-01	-8.705E-01
IY= 6	-4.504E-02	7.386E-01	-2.282E+00	-3.930E-01	-3.343E-01
IY= 4	1.099E-02	6.623E-03	-7.621E+00	2.912E-01	1.797E-01
IY= 2	1.779E-01	8.044E-01	-1.555E-01	1.231E+00	6.733E-01

VALUES OF U2

IY	IX=1	3	5	7	9
IY=10	-1.277E-01	9.524E-01	-6.137E-02	-1.416E+00	-1.167E+00
IY= 8	-7.131E-02	6.494E-01	3.010E-01	-8.277E-01	-6.716E-01
IY= 6	-4.214E-02	5.342E-01	5.744E-01	-2.352E-01	-1.530E-01
IY= 4	3.679E-02	6.331E-02	2.811E-01	4.494E-01	3.093E-01
IY= 2	3.377E-01	1.348E+00	6.056E-02	1.283E+00	9.713E-01

VALUES OF V1

IY	IX=1	3	5	7	9
IY=10	0.	0.	0.	0.	0.
IY= 8	-5.439E-02	4.373E-01	-9.174E-01	-2.050E-01	1.813E-01
IY= 6	-1.216E-01	8.358E-01	-1.310E+00	-3.347E-01	3.297E-01
IY= 4	-1.655E-01	1.152E+00	-1.738E+00	-2.424E-01	4.349E-01
IY= 2	-1.310E-01	4.564E-01	7.196E-01	1.917E+01	3.733E+02

VALUES OF V2

IY	IX=1	3	5	7	9
IY=10	0.	0.	0.	0.	0.
IY= 8	-2.353E-01	2.415E-01	-9.486E-01	-2.603E-01	1.893E-01
IY= 6	-3.337E-01	5.215E-01	-1.467E+00	-4.033E-01	3.190E-01
IY= 4	-4.321E-01	5.334E-01	-1.741E+00	-4.144E-01	3.649E-01
IY= 2	-3.619E-01	3.247E-01	5.647E-02	-3.443E-01	2.358E-01

IPSA 1981	36/37	DISCUSSION OF THE PIPE-BEND RESULTS

- They were easy to obtain.
- Runs with higher and lower friction factors were also made.
- Divergence occurred in some low-friction runs, and was cured by reduction of time step.
- Even with high interphase friction, large-amplitude 'sloshing' motions developed.
- Computer times were modest (< 10 s for the run described, on CDC 6400).
- Comparisons with experiment (if they were possible) would be pointless at this stage; for the interphase-friction formula (∝ r R (u - U) etc) was simply guessed.

IPSA 1981	37/37	CONCLUSIONS

- Regarding 1D results:
  - Wave-propagation effects in 2-phase mixtures, with and without stratification, take many forms.
  - Computed results appear plausible and provide insight.
- Regarding 2D results:
  - Sedimentation in a curved duct gives rise to large-scale and large-amplitude motions.
  - Computer results, surprising at first, now seem plausible.
- Regarding IPSA:
  - It is a robust and fairly economical solution procedure, both in single- and two-pressure versions.
- Regarding computer codes:
  - Numerical modellers who are more interested in physical results than in computer programming may find it convenient to attach their special subroutines to the PHOENICS Code System.

PHOENICS 1981	1/20	A GENERAL-PURPOSE COMPUTER PROGRAM FOR MULTI-DIMENSIONAL ONE- AND TWO-PHASE FLOW by D BRIAN SPALDING

CONTENTS

PHOENICS 1981	2/20	EIGHT INGREDIENTS OF A COMPUTER SIMULATOR FOR FLUID-FLOW PROCESSES

1. Finite-Domain Equations of the Conservation Equations ✓
2. Means of Solving them Economically ✓
3. Auxiliary Relations Representing 'Models' of Complex Phenomena (Turbulence, Chemical Kinetics, Radiation) ✓
4. Geometric Data Describing Confining Walls ✓
5. Initial Values of Dependent-Variable Fields ✓
6. Boundary-Condition Information, Defining Particular Processes ✓
7. Thermodynamic and other Property Data of Fluids ✓
8. Computer Code Putting All the Above Together ?

PHOENICS 1981	3/20	WHY CREATION AND MAINTENANCE OF A FLOW-SIMULATING COMPUTER CODE IS DIFFICULT

1. Differential Equations are numerous; and real physical phenomena entail numerous and complex boundary conditions and auxiliary relations.
2. Non-linearity of equations necessitates guess-and-correct solution procedures, which can easily diverge.
3. Even if convergence can be attained, slow convergence causes prohibitive expense.
4. Even when a program exists, and solves successfully the bench-mark problems, no guarantee can be given that it will solve others.
5. Even a fully satisfactory program can be caused to fail by well-meant misuse, and so discredited.
6. If the Fortran is accessible, small but disastrous changes can be made, without detection.

PHOENICS 1981	4/20	THE QUESTION OF RESOURCES

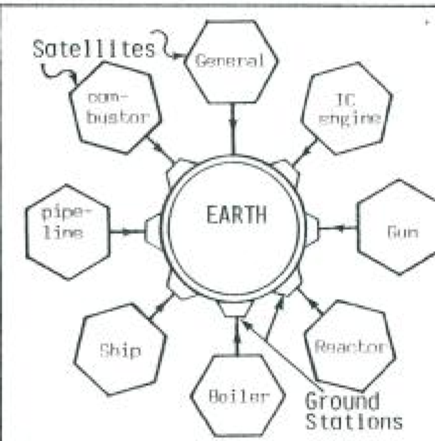
- All the difficulties can be overcome; but an experienced team, operating a refined code-management system, is needed.
- Few successful fluid-mechanics codes come into existence without tens of man-years of development effort.
- Even after creation, a maintenance team is needed to repair damage, and prevent deterioration.
- Resources can seldom be found which will fund such efforts for special-purpose fluid-flow simulators (Exception: nuclear safety, TRAC, RELAP).

PHOENICS 1981	5/20	CONCLUSIONS ABOUT FLUID-FLOW COMPUTER-CODE POLICY

1. Expectations that individuals, or under-funded teams, can produce reliable computer codes, suitable for use by others, should be given up.
2. The essential unity of all fluid-dynamic phenomena (in respect of differential equations) should be exploited, by creating a general-purpose fluid-flow simulator.
3. Resources deriving from many application areas can then be 'pooled'.
4. The inner mechanisms should be protected from interference.
5. The user's need to check, adapt and extend must be satisfied.

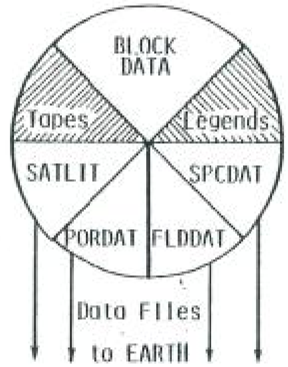
PHOENICS 1981	6/20	PART 2: THE PHOENICS SOLUTION STRUCTURE

- PHOENICS is a system of computer codes (satellites), all connected with a common core (EARTH).
- 'Ground-Station' subroutines may assist EARTH to simulate special plant and processes.
- Most users work with Satellites alone; skilled users make changes in Ground Stations; EARTH is 'out of bounds' to all.



PHOENICS 1981	7/20	A PHOENICS SATELLITE SHOWING THE SUBROUTINES IT CONSISTS OF

- BLOCK DATA enters 1334 data items, arranged in 44 groups. All have 'reasonable' defaults, so users need make few settings.
- PORDAT sets up non-standard geometries by use of 'porosity factors'.
- FLDDAT provides initial fields of dependent variables.
- SPCDAT provides special data needed by Ground Stations.
- SATLIT organizes the calculation and sets up a series of runs.

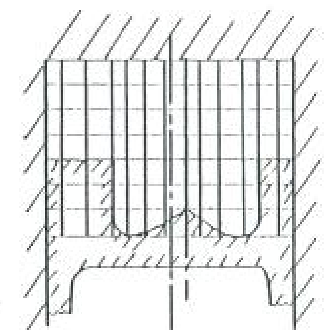


PHOENICS 1981	8/20	SOME SAMPLE BLOCK DATA ENTRIES, SUITABLE FOR AN AXISYMMETRICAL COMBUSTOR, BURNING COAL IN AIR

```
DATA  STEADY, CARTES, PARAB/.TRUE., 2..FALSE./
DATA  NX, XULAST, XFRAC / 1, 3.14159, 30 . 1.0 /
DATA  NY, YVLAST, YFRAC / 30, 0.06, -30.0, 0.002, 28 . 1.0 /
DATA  NZ, ZWLAST, ZFRAC / 50, 0.50, -50.0, 0.01, 28 . 1.0 /
DATA  ONEPHS / . FALSE /
DATA  SOLVAR (12), SOLVAR (13) / 2 . .TRUE. /
(This switches on the k and ε equations).
DATA  FIINIT (7), FIINIT (8) / 2 . 0.1 /
(This sets the axial velocities of the two phases to
 0.1 m/s at the start).
DATA  IMDOT, CMDOT / 2, 3.E5 /
(This chooses an interphase transport law, and sets a
 rate constant).
```


PHOENICS 1981	9/20	WHAT PORDAT DOES

- PORDAT determines what proportions of all cell volumes and areas are accessible to fluid.
- It can enlarge these, as well as diminishing them, so as to fit a grid to curved boundaries.
- It provides values at the start; but an EXPAND facility permits stretching and contraction during computation, needed eg for I C engines.



Example of I C-Engine Cylinder with offset bowl in piston

PHOENICS 1981	10/20	THE GROUND SUBROUTINE

- GROUND is a user subroutine, supplied in a standard format, communicating with EARTH by:
  (1) being called from EARTH at specified points in the computation cycle;
  (2) getting values of variables from EARTH by way of a GET subroutine;
  (3) communicating new values of these (or other) variables to EARTH by way of a FIX subroutine;
  (4) printing any of the values by way of a PRNSLB subroutine (or otherwise).
- GROUND has a COMMON identical with that of the satellite; but it does not have the main EARTH COMMON.

PHOENICS 1981	11/20	THE GROUND SUBROUTINE (continued)

GROUND is a 'working platform' on which the user can erect coding, in his own style, which will:

- provide non-standard thermodynamic or transport properties for the fluids;
- provide non-standard chemical-kinetic or interphase transport laws;
- replace EARTH's built-in turbulence model (k - e) by any other;
- provide non-linear flow-resistance laws;
- represent inter-variable sources and sinks, eg representing the presence of 'guide vanes'.
- provide special boundary conditions, eg concerning impingement of particles on a wall, and adhesion to it.

PHOENICS 1981	12/20	THE GROUND SUBROUTINE (concluded)

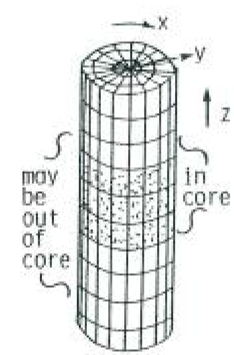
- GROUND can be used as an independent check on EARTH, by way of coding which performs again what EARTH is supposed to be doing (eg getting cell-by-cell balances).
- GROUND may incorporate novel equation-solving sequences, perhaps rendering those of EARTH redundant.
- Users of GROUND need therefore not be held back by any limitations in EARTH's built-in capabilities, or forced to take any of EARTH's computations on trust.

PHOENICS 1981	13/20	THE EARTH PROGRAM: WHAT IT DOES

- EARTH sets up a 1D, 2D or 3D grid, cartesian or polar (other options exist).
- EARTH solves either steady or transient problems.
- EARTH solves for up to 25 named variables, including:
  - 3 velocity components of first phase;
  - 3 velocity components of second phase;
  - enthalpies of 1st, 2nd and 3rd phases;
  - 3 radiation-flux sums;
  - 4 concentrations;
  - k and e of the first phase.
- EARTH embodies fully-implicit formulations.
- EARTH uses the SIMPLEST and IPSA solution procedures.

PHOENICS 1981	14/20	THE EARTH PROGRAM: HOW IT DOES IT

- Computational cells are visited in a repeated series of 'sweeps', from low z to large (only 1 sweep is needed for parabolic problems).
- Dependent-variable values over a 'slab' of cells are up-dated at each visitation, by a combination of Jacobi and simultaneous-solution techniques.
- Cell-wise and overall residuals in finite-domain equations are monitored. Iteration is terminated when they are smaller than pre-set quantities.

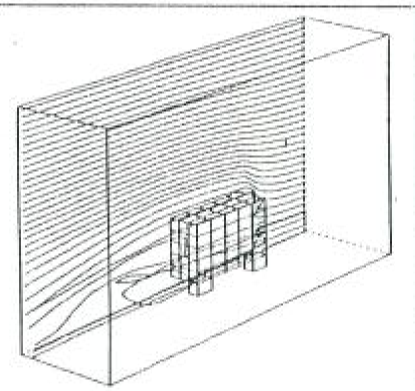


PHOENICS 1981	15/20	SOME FACTS AND FIGURES

- PHOENICS was developed on Perkin-Elmer mini-computers 7/32 and 32/20 (32-bit words).
- PHOENICS comprises approximately 11,000 FORTRAN statements in ~60 subroutines.
- Without Ground Stations, EARTH + Satellite takes about 200k words of storage on the Perkin Elmer 32/20.
- PHOENICS is running on CDC, IBM, Univac, Cray and ICL equipment.
- Particular versions (ie Satellite-GROUND combinations) in current use simulate:
  - 1D : Pipe-line flows, two-phase and compressible.
  - 2D : Flow in cascades; some nuclear-reactor flows.
  - 3D parabolic : Ship wakes; exhaust plumes.
  - 3D elliptic : Nuclear reactors; reciprocating engines.

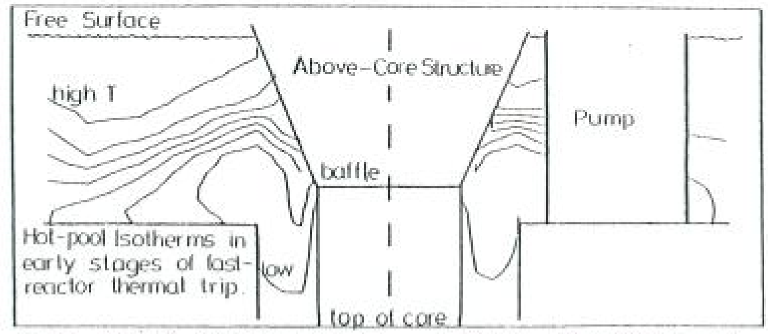
PHOENICS 1981	16/20	EXAMPLES OF PHOENICS COMPUTATIONS: 1. FLOW AROUND A VEHICLE

Grid:	NX = 8 NY = 12 NZ = 20
Phases:	One
Transience:	Steady
Model:	Laminar
Computer Time:	1800 sec cpu time



Work by: C Aldham (1981)
Plotting by GRAFFIC

PHOENICS 1981	17/20	EXAMPLES OF PHOENICS COMPUTATIONS: 2. FLOW IN A FAST-BREEDER NUCLEAR REACTOR



Hot-pool isotherms in early stages of fast-reactor thermal trip.

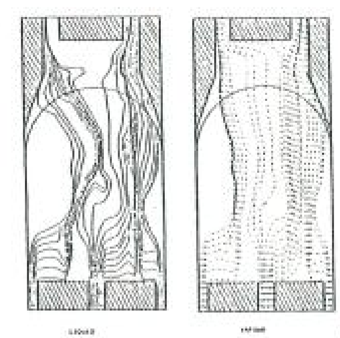
Grid: NX = 15, NY = 15, NZ = 15
Phase: One; Transience: Unsteady
Model: k-ε
Computer Time: 1000 sec cpu time per sec real time

Work by: P J Phelps, N C Markatos THCM 1981

PHOENICS 1981	18/20	EXAMPLES OF PHOENICS COMPUTATIONS: 3. FLOW IN NUCLEAR STEAM GENERATOR

Grid:	NX = 1 NY = 14 NZ = 18
Phases:	Two
Transience:	Steady
Model:	Fixed Viscosity
Computer Time:	3600 sec cpu time

Work by: M R Malin (1981)
Plotting by GRAFFIC



PHOENICS 1981	19/20	PART 3: THE PRESENT POSITION

- PHOENICS is regarded by its originators as ready for experimental release to selected users.
- It can be accessed through major computer networks.
- Economy is such as to make 3D calculations with over 5000 cells and 15 differential equations practicable (for rich users). Most 1D and 2D calculations are regarded as inexpensive by users.
- Automatic divergence-preventers have not been built in; so users may need to adjust 'control knobs' to procure convergence in some cases.
- Adequacy of problem-adaptation and result-checking capabilities are for users to judge.

PHOENICS 1981	20/20	CONCLUDING REMARKS: THE ARGUMENT RECAPITULATED

- Reliable fluid-mechanics computer codes are VERY DIFFICULT to create, and to maintain.
- Resources rarely suffice to allow the difficulties to be overcome for a single-purpose code.
- However, the laws of fluid mechanics, heat-transfer, etc being universal, a code embodying them in a general form can be created.
- Resources can then be gathered from many diverse applications of the single general-purpose code.
- PHOENICS is the first result of an attempt to do this.